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What Price Enrichment?*

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Many words have invaded our national educational domain in recent times like fifth columnists to entice us, disarm us, and in every way cripple us in our national preparedness campaign against ignorance and inefficiency. One of these is the word *enrich*, with its breed of derivatives like enriched, enrichment, etc. It is a sweet sounding word. When rolled under the tongue of an enthusiastic representative of "Progressive Education," it has had marvelous effect. We have become attached to it and have welcomed it into our educational literature. Webster gives two definitions: (1) To increase the value of, and (2) to adorn or make more ornamental. We have had enriched curricula, enrichment programs, and what not. Some produce increased values. Some just adorn. It is my purpose to point out what I consider some "disenriched" and disastrous consequences of our short sighted friendliness with this seductive invader.

In too many places more emphasis is being placed on the band and the football team than on all other school activities together. Fortunately the football fever is chilled by the winter snows, but the band epidemic rages on through the year. Even those who haven't the musical ability to play can attend a course for drum majors and lead the parade of the musicians, tripping over their poorly twirled, falling batons as their fond, adoring parents watch from the sidelines, hoping the weather isn't too cold for daughter's bare knees. These students have taken on another extra-curricular activity. Their educational program has been enriched.

Other examples could be cited, and some have doubtless arisen in your mind. I shall not take time to mention them.

How the enrichment program has invaded the realm of mathematical instruction can easily be seen by examining some modern texts. In a text on applied mathematics published in 1933 for ninth grade high school students, we find three chapters covering seventy-four pages on means of communication, postal information, and means of travel. This is a little more than one-sixth of the entire text. In these three chapters there are less than one hundred exercises in computation, restricted to operations with integers, decimals, and fractions. Allowing for exercises having several parts, the total number of individual operations might be about three hundred. From the lack of difficulty of the exercises we can estimate that about eight to ten hours of study would be required in solving them. The rest of the material is made up of pictures of Pullman berths, air-

* Speeech delivered at the Baton Rouge meeting of the National Council of Teachers of Mathematics in December 1940.

planes, and railway schedules, with instructions on everything from how to buy a railway ticket to what you ought to give your postman for Christmas. One whole page is used to instruct the student in telephone courtesy—how not to jiggle the hook and not to say "Hello" when you answer the phone. Here is a sample of the many non-mathematical exercises: "Repeat the following telephone numbers correctly:" (followed by several sample telephone numbers such as "Wabash 4951"). Others are: "List in the telephone directory at your home the names and telephone numbers of persons and firms that the family often calls." "In what way is it harder to carry on a conversation over the telephone than face to face?" "Tell what you would do if you saw the house next door on fire." One and one-third pages are taken up with instructions (accompanied by diagrams) on how to fold a letter and put it in an envelope. Here is a sample (in two successive exercises) of the mixture of mathematical and non-mathematical material:

Ex. 3. How much must be given the postmaster in exchange for a money order for $87.69? For $26.47? For $56.87? For $65? For $80? For $93.68?

Ex. 4. Write an application for a money order for $5 for a fountain pen from Marshall Field & Co., Independence Square, Philadelphia, Pa.

Two pages later we find twenty exercises like the following: "What is the purpose of the Dead Letter Office?" Following a full page reproduction of two bus line timetables is a series of exercises in simple subtraction based on finding the distances between stations in the timetable. No one would object to these, that is, for fourth or fifth grade students, but why nine questions on the next page like this one? "Give two or more reasons for the rapid growth of bus transportation."

In another text published in 1938 for junior high school students we find a unit on banking containing thirty-two pages. The first twenty-one pages describe in detail everything from the federal Reserve Act of 1913 down to how to endorse a check, and not a single exercise or problem can be found. The total number of exercises involving mathematics in the entire unit is eighteen, and they are restricted to simple operations on integers, simple interest, and percentage. By no stretch of the imagination could a student use more than five hours of work doing all of the mathematical exercises. There are thirty-six questions in the chapter like "What is an overdraft?", "What is a certified check?", and "Why does a bank send monthly statements to its customers?" At the end of the unit is a "Mastery Test." It has forty-four questions on business procedure, including nothing quantitative, and eighteen on arithmetic.

In contrast to the two books just mentioned, we shall consider now a text published in 1937 called *Socialized General Mathematics*. It was written by a well known and reputable author of mathematics texts. He also introduces modern material illustrating ordinary life experiences to motivate the work of the student. Whereas one of the other books uses up pages and pages to present railroad travel, this one in eleven lines states clearly the method and cost of traveling on a railroad. Then there are eighteen drill exercises in computation which require finding the cost of various railroad journeys. A similar situation exists in the section on traveling by airplane. There is a picture of an airplane, a timetable, and a page and a half of exercises in computation. Everywhere the social setting is *adequate*, though *minimized*, and the mathematics is *evident* and *emphasized*. This is as it should be.

Too many teachers of mathematics have been led astray by this so-called "enrichment" of mathematics courses. Its growing importance reminds me of a poem I learned when a boy in an old fashioned school reader. You may recall the poetic fable of the camel with the cold nose who appealed to a workman for entrance into the shop for warmth. I quote part of the poem describing the result.

"Since no denial word was said,
In came the nose, in came the head.
As sure as sermon follows text,
The long and scraggy neck came next;
And then, as falls the threatening storm,
In leaped the whole ungainly form.

Aghast the owner gazed around,
And on the rude invader frowned,
Convinced, as closer still he pressed,
There was no room for such a guest;
Yet more astonished heard him say,
"If thou art troubled, go away,
For in this place I choose to stay."

This "socialization," this "children's economics," this "enrichment from life experience" is crowding out of the curricula of the grades and junior high school civilization's master workman, arithmetic. It has not only reduced the proportion of time in the curriculum given to mathematics but, as indicated by the examples I have given, has reduced the amount of time in the mathematics courses given to mathematics. Having virtually crowded out arithmetic, now the nose and head of the beast are already poking through the door of our algebra and geometry. Unfortunately the "enrichment" beast is not ungainly so we have not only not been alarmed, but we have encouraged his entrance.

I shall make no argument against teaching economics, sociology, psychology, and various other social subjects to high school students. My plea is that we do insist that such material be taught in courses so designated and not disguised as mathematics. In order to "integrate" the curriculum, we may need to introduce "socialization" into our mathematics. Certainly we wish to teach our students where mathematics can be used. But if we must teach the social sciences too, let us demand that a corresponding extra amount of time be given to the new hybrid course so that the total amount of actual mathematics shall not be reduced.

I promised at the outset to point out some "disastrous consequences" of this encroachment on mathematics. Here they are:

For three years the University of Oklahoma has been giving a placement test in mathematics to all freshmen. About 1300 freshmen take the test each September. Upon inquiry I discovered that another mid-western state university is doing a similar thing. I also got some recent results on student proficiency secured by an eastern university. The responses to my inquiries at a western and southern state university failed to yield any quantitative data but seemed to confirm the belief that high school graduates show less and less proficiency in mathematics as the years go by. Hence, these embarrassing results that I am about to present are not peculiar to Oklahoma. In fact, not all of these results are taken from the Oklahoma tests.

The tests we give to the high school graduates do not cover comprehensively any previous course in mathematics and cannot be considered in any sense as entrance examinations. They are far too simple. They do try to measure proficiency in the fundamental processes of arithmetic; the understanding and use of the very simplest operations of algebra; the knowledge of the most useful geometric relations in space; and most of all the ability to do the quantitative thinking so badly needed in most of the university courses. We think the samples of quantitative thinking and the manipulative processes involved in our tests are those needed also by the average citizen as well as by a prospective college student.

The results that I shall now report are taken from tests at different places in different years and in each case where percentages are given, more than 1000 students tried to solve the problem. Sixteen per cent could not multiply 2.65 by 32.4 correctly; 11% could not add $\frac{2}{3}$ and $\frac{1}{7}$, 3% getting 3/10 for the answer; 10% could not multiply .2 by .3 correctly; 36% could not get the correct answer to $3\frac{1}{2}\times 4\frac{1}{4}$; 57% could not express $\frac{7}{8}$ as a decimal fraction, 20% omitting it and 37% getting it wrong; 38% failed to get the correct answer in dividing 1276.4 by 1000; and

71% could not express 32.5% as a common fraction. In the realm of simple algebra we found that 34% could not correctly answer "If a and b are two numbers, write an expression which means the sum of their cubes," and on another test 40% failed on this one, "Express in algebraic symbols: twice the sum of two numbers, x and y, is equal to one-half their product." More than 34% failed to solve the equation $2x^2 = 18$, while 23% could not solve $2x + \frac{1}{2} = 5$, and 45% failed to solve $4/x = 12$. Forty per cent of those who attempted to factor $a^2 - b^2$ failed to do it, and 75% could not "write in symbols the total number of feet in x yards and y inches."

On one test the students were asked, "If an airplane travels at the rate of 192 miles per hour, how far does it go in 45 minutes?" Some got more than 192 for the answer and 15.4% missed it. They were also asked, "How long must I wait if I wish to board a train due at 15 minutes past 11 and it is now 20 minutes past 8 o'clock?" One out of every seven could not answer correctly. In response to the question, "If 10 men can dig a ditch in 7 days, how long would it take 5 men to dig it if they all worked at the same rate?" one out of every three missed the answer and one out of every four said $3\frac{1}{2}$ days.

Recently there came to me from a citizen of Oklahoma the following letter:

"Dear sir: Would you in your spare time work this problem and return the answer to me? I want to know what .008 divided into $17.24 will be in dollars and cents. Hoping to hear from you soon, . . ."

Believe it or not, people occasionally need to do computations like this.

If this writer had asked some of the high school graduates of Oklahoma the question, the chances are, according to our records, that only 2 out of every 3 could have told him the answer, for about $\frac{1}{3}$ of those we tested could not do division of decimals correctly. The records I secured from two other states show, for the same kind of question, 28% and 37% wrong, respectively.

The propagandists for enriched mathematics courses in progressive education will immediately counter with two statements. First, the crowding in of enriched material is not the sole cause of this lack of proficiency in high school graduates, and second, we cannot educate all high school students as if they were going to college. To the latter statement, I reply that the problems on which were made the poor records are such as should be solved by the average citizen; for example, like the one in the letter just cited. These tests are not designed as entrance examinations into college mathematics at all. Many of the students who take them do not take mathematics in college. The teachers of courses in the School of Business, in Economics, in Sociology, and, of course, in all of the sciences, do not want these students who are weak in such fundamental things. For the same reason the citizen who is to take such a place in the community as befits a high school graduate and meet the everyday competition of a modern scientific world such as ours will find himself handicapped if he cannot do these things. To the first question my answer is that it is true that other factors help cause these disgraceful results, and they should also be eradicated. There is not enough time to consider them here. I merely insist now that it certainly does decrease proficiency in necessary quantitative operations and quantitative thinking to crowd out of the meager time now allotted to mathematics so much of the drill that should be given in these things in order to replace it by such teaching as how to fold a letter or why a bank sends monthly statements to its customers.

Naturally there is a difference of opinion as to what is the most valuable subject matter in arithmetic. Is it perhaps that related matter from enriched community experience wherein the processes of arithmetic are used? I have said, I have published, and I still insist that *the most important and most practical thing to be learned in arithmetic is a clear understand-*

ing and efficient use of the fundamental operations of addition, subtraction, multiplication, and division on integers, common fractions, and decimals. These are the parts of arithmetic used daily by the average person and used more than anything else learned in the whole school curriculum except reading and writing. Any course is deficient that neglects to emphasize these things and fails to provide sufficient material for drill on these operations.

The same general principle applies to algebra. *Problems* are solved by *equations* and *algebraic technique* is necessary. Furthermore, in geometry no pictures of geometric tile patterns or beautiful designs of art windows, or even scores of exercises in constructive drawing, will fill the need of youth to be taught the pattern of logical thinking that can be presented by properly teaching the subject matter. Let us beware of any enrichment that does not contribute directly to the student's comprehension of the applications of geometry to other fields or to the patterns of logical thinking similar to geometry to be found in all fields of mental endeavor.

Such glaring weaknesses among our high school graduates as I have presented have also been found in other places by other investigators. They will be found in greater numbers in time to come unless we can restore a normal, healthy respect for the manipulative processes of mathematics, skill in which can be had only through drill. We do need motivation. We do need to make contact with the situations in which processes are used. But "what doth it profit a man" if he have the very best appreciation of the possible use of mathematics and a perfect understanding of what principles to use in a given situation if his manipulative skill in using those principles is so poor that he cannot get the answer? Through many centuries mathematicians have discovered and refined those mathematical methods and rules demanded by a growing civilization. Let us not in a moment of weakness barter the pupil's birth-right of a fundamental understanding of these assured values of race experience for a mess of pottage of modern, possibly temporary, community commercial practices. Against too much encroachment of so-called enrichment of our courses in mathematics we must stand firmly in order that the power of accurate computation, quantitative thinking, and logical reasoning may not perish in our nation.

The Famous Sugar Plum Problem

From Stockton's Western Calculator, 4th Edition 1851. First Edition 1818–19

Contributed by L. LELAND LOCKE, *Brooklyn College*

A, IN A scuffle, seized on 2/3 of a parcel of sugar plums; *B* catched 3/8 of it out of his hands, and *C* laid hold on 3/10 more, *D* ran off with all *A* had left except 1/7, which *E* afterward secured slily for himself: then *A* and *C* jointly set upon *B*, who in the conflict let fall 1/2 he had, which was equally picked up by *D* and *E*: *B* then kicked down *C*'s hat, and to work they went anew for what it contained; of which *A* got 1/4, *B* 1/3, *D* 2/7, and *C* and *E* equal shares of what was left of that stock: *D* then struck 3/4 of what *A* and *B* last acquired out of their hands; they with difficulty recovered 5/8 of it in equal shares again, but the other three carried off 1/8 apiece of the same. Upon this they called a truce, and agreed, that the 1/3 of the whole, left by *A* at first, should be equally divided among them—How much of the prize, after this distribution, remained with each of the competitors?

The least number satisfying the conditions is 26880.

The Improvement of Mathematics Teaching in the Secondary School

By Hob Gray
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A teacher of trigonometry was once asked in class why it was necessary to be so painstakingly accurate in working the problems given in his assignments. To illustrate the need for accuracy he related the story of two engineers who had just completed the building of a bridge and were going out the next morning to see how it looked when completed. On arriving at the spot they found the bridge a tangled, twisted mass far down in the river bottom. One of them turned to the other and said despairingly, "See there, Bill, we got that decimal point wrong."

Today, when we examine the records of college and university freshmen with respect to ability to work mathematics, we find that somewhere in the secondary school (or earlier) a "decimal point" in teaching has been misplaced.

There are many views on this question of what is wrong with mathematics teaching and mathematics courses in the high schools. To begin let us consider the generally accepted defects in secondary-school mathematics teaching. It seems that almost everywhere the high schools are houses divided against themselves with respect to what should be taught in mathematics courses. This division of purpose arises from the fact that there are two distinct types of high school students: a) those who are going to college, and b) those who are not going to college. The two groups of students require types of mathematics that are considerably different. The first-mentioned group needs to learn that which will enable them to master the more advanced phases of the subject; the second group needs that which will be useful in the life of the consumer.

Another difficulty with our present-day system is that curricula are being set up merely on the basis of personal point of view without real qualifications on the part of those determining the curriculum. Frequently the college or university courses which the high school teacher has taken have not "sold" him on mathematics; that is, the all-pervasiveness of mathematics has not dawned on the college student, and he in turn does not instill a full appreciation of mathematics in his pupils. In the words of W. D. Reeve, speaking at the recent convention of the American Association of School Administrators, " . . . we must also open up mathematics, so to speak, so as to give each child a chance to see what the subject means, what his likes and dislikes are and in which direction his future interests lie. I feel perfectly sure that we must teach not so much mathematics but more about mathematics."

Professor Reeve[1] points out that too many teachers have a weak academic background, both mathematically and educationally. Some of the causes for inefficient and unsatisfactory teaching include:

1) Lack of Personality
2) Professional apathy
3) An insularity of mind
4) The need for a personal philosophy of education

Mr. Alan D. Campbell of Syracuse University[2] found from his study of the

[1] As quoted by W. L. Schaaf in an article entitled "The Education of Mathematics Teachers" in *Natl. Math. Mag.*, Vol. XIII, No. 2, p. 86 (November, 1938).

[2] Campbell, Alan D., "Some Weaknesses in Mathematical Training" in *Natl. Math. Mag.*, Vol. XII, No. 7, p. 348. (April, 1938).

problem that the stifling of originality and curiosity was too much in evidence.

Finally we might say that textual material is presented in the deductive rather than the inductive method; the natural way in which mathematics has developed is completely reversed so that the pupil studies the subject in exactly the opposite way from that in which the subject was originally grasped.

Having obtained a "bird's eye" view of what is wrong in our present set-up, we should turn to possible ways of remedying the difficulties. W. D. Reeve[3] has given a suggested program for mathematics beginning with the seventh grade and extending up through high school. In the seventh grade, arithmetic should be built around problems of the home, daily purchase, industry, banking, farming, and the like; informal geometry should be begun, occupying about one-half of the year. The study of algebra should likewise be started and introduced, perhaps, under the heading of the geometry of size. In the eighth grade there should be a thorough review and careful extension of the seventh grade material. Plenty of time should be given to algebra (as much time as is given to arithmetic and geometry). In the ninth grade, algebra should continue up to and including the study of quadratics; however, the study of quadratics should not go beyond graphing. Possibly a unit of numerical trigonometry should be included in order to continue inculcating ideas of "spatial character and their quantitative relations." A very important recommendation for the ninth grade is that "demonstrative geometry should be required of all pupils except a very few." From the ninth grade on, the work should be elective. Mr. Reeve also suggests that in the senior high school the work be of a general mathematics nature. The fact that the material is elective makes it possible for the student to have his way to some extent. It is at this point in his schooling that he is very likely to develop a taste for certain types of mathematics. In the tenth year the unifying thread should be geometry. The eleventh year should be made up of algebra, trigonometry, and the fundamental elements of analytic geometry. One of the most interesting recommendations is that the fundamental ideas of the Calculus should be presented in the twelfth year. Further, somewhere in the senior high school it might be well to include a unit of work on social-economic arithmetic.

An impressive overtone in the call for improved mathematics teaching is that the basic program of mathematics should center around living; i.e., the things that should be stressed are income, shelter, food, social security, and medical care. H. B. Risinger[4] of Rutgers University inclines to this view by listing three responsibilities of mathematics: 1) mathematics for personal use; 2) mathematics for social use; and 3) mathematics for vocational use.

In accord with the progress of technology, it is being strongly recommended that training should be given leading toward an understanding of precision; that is, students should be instructed so as to enable them to understand the use and interpretation of blue prints, diagrams, charts, etc.

A special commission appointed by the Mathematical Association[5] has recommended that the teacher should have a minimum of 24 semester hours in pure mathematics. Thirty-six hours of work was considered desirable. Also it was recommended that the number of hours in related work be 21, and the desirable number be 33. About 1933 the joint data of the American Committee of the International Commission[6] and the National Survey of

[3] Speaking at the convention of the American Association of School Administrators and Allied Organizations at St. Louis, Mo., Feb., 1940.

[4] Speaking at the convention of the American Association of School Administrators and Allied Organizations at St. Louis, Mo., Feb., 1940.

[5] W. L. Schaaf, "The Education of Mathematics Teachers" in *Natl. Math. Mag.*, Vol. XIII, No. 2, p. 84. (November, 1938).

[6] *Ibid.*, Vol. XIII, No. 2, p. 85. (November, 1938).

Education of Teachers implied the following standards:

a) Introduction to educational concepts, three semester-hours.

b) Psychology, measurements, and other courses in educational theory, nine semester-hours; the teaching of mathematics (professional treatment of materials), six semester-hours; observation and practice-teaching, three semester-hours.

The Commission for the Training and Utilization of Advanced Students of Mathematics recommended a "one-year course in methods of teaching and practice teaching in secondary mathematics, together with any distinctly pertinent material concerning educational measurements and other content from educational theory (ten semester-hours)."

In the Davey Junior High School in East Orange, New Jersey, and in Rutgers Demonstration School (New Brunswick, New Jersey) there has been substituted for the traditional content of courses material of more social import. Professor Risinger in reporting on this work says, " . . . ever increasing amicable relations between the school and community have developed; the pupils have outstanding ability to carry on research; and they demonstrate greater competence in dealing with vital social problems."

Thus we see that defects lie mainly in textual material, preparation of teachers, and general philosophy with respect to the *raison d'être* of mathematics in the schools.

In a recent questionnaire[7] there appeared one query which bears directly upon our subject. This query ran as follows: "Shall mathematics be taught to the academically gifted few, or shall some sort of mathematics be taught to all whose presence is accounted for through the junior college level?" There was considerable variation in the replies to this question, but one of the most interesting answers was: " . . . It is strictly up to mathematicians as a whole to make mathematics really and obviously worth while to all students. I would make drastic changes in content. . . . When we as a whole make our work really worth while to all students we will not need to argue the question. . . . But that is still a long story."

After considering the arguments, both pro and con, concerning the improvement of mathematics and the desirable philosophy that should serve as a guide, it seems that we should select our best students and encourage them as much as possible to go on with work in more advanced mathematics. But in no way should the other types of students be excluded. In other words, the keynote of a saner program of mathematics teaching should be: *Selection rather than elimination*

[7] Seidlin, Joseph, "The Place of Mathematics and Its Teaching in the Schools of this Country" in *Natl. Math. Mag.*, Vol. XI, No. 3, pp. 150, 151 (December, 1936).

We have been asked to announce a series of Curriculum Bulletins which are being issued in mimeographed form by the Curriculum Laboratory, University of Oregon, Eugene, Oregon, of which Professor Hugh B. Wood is the Director. Of particular interest to mathematics teachers is No. 11, "Mathematics, A Study Guide for Teachers," prepared by Vernon E. Kerley, and utilizing some materials prepared by Mr. Wood. The guide "provides a basis for directed study of some of the problems of the mathematics curriculum, and may be obtained for 25¢." Other Bulletins which have been received at this office are:

No. 4 (Revised), FREE AND INEXPENSIVE MATERIALS, An Annotated Bibliography of Bibliographies of Sources of Pamphlets and Other Teaching Aids Obtainable Free or at Small Cost . . . Elizabeth Findly. 25¢

No. 17, AN INDEX TO VISUAL AND AUDITORY AIDS AND MATERIALS, An Annotated Bibliography of Bibliographies and Sources of Audio-visual Aids for Rent, Purchase, or Free Distribution . . . Elizabeth Findly. 35¢

No. 24, PRICE LISTS OF INEXPENSIVE TEACHING MATERIALS, Hugh B. Wood. 15¢

How the National Council of the Teachers of Mathematics May Serve Negro Teachers and How They May Serve the Council*

By Ethel Harris Grubbs
Washington, D.C.

The assigned subject—"How the National Council of Teachers of Mathematics may Serve Negro Teachers and How They may Serve the Council"—lends itself to development from a highly theoretical viewpoint. This paper, however will consider the subject first, as a plain, practical projection of the problems encountered by Negro teachers of mathematics in schools for Negroes; second, as a series of recommendations of cooperative action between Negro teachers and the Council.

Upon the assumption that a council is a body of persons whose main function is to consult, deliberate, and advise on matters of common interest, pertaining to the welfare of its members, this paper is presented for your candid consideration and criticism. It becomes necessary, then to establish certain fundamental data, to draw upon past experiences, and to suggest means for correcting the problems presented.

(1) It is of more importance to discuss means of overcoming the environmental handicaps of the Negro child than to consider the frequently heard but unproved assumption of his mental inferiority or of the need of a special adaptation of the mathematics program because of peculiar racial needs.

(2) It is of more importance to evolve remedial measures as the result of observation and study rather than as promiscuous offerings of theoretical panaceas.

(3) Further, it is of vital urgency that the functional teaching of mathematics shall be viewed as one phase only of the problem, that a well-grounded knowledge of mathematics shall be considered as necessary in the daily life of man as a consumer, a worker, a citizen.

This discussion of how the Council may serve Negro teachers of mathematics is limited to the Negro teachers of mathematics in schools for Negroes. Special stress will be placed upon those schools in which no attempt is made to provide for the maximum amount of training for Negro children in mathematics as provided by the state course of study and upon those which do not meet the curricula requirements of the state in which they are located. Inherent in these situations are some primary difficulties. Among these are: (1) the limited professional preparation of the teacher; (2) the short school term; (3) poor attendance on the part of the pupil; (4) inadequate physical equipment; (5) low salary scales; (6) heavy teaching loads; (7) and, the lack of community cooperation in Negro school problems.

The professional preparation of Negro teachers is a subject of many paradoxes. Because of poor training, salaries are low; because of low salaries, opportunity for training and professional improvement is limited. Because of varying standards of certification the level of certification is low in many communities, and vice versa. Sometimes low standards of living is given as a cause for low salaries, when it is quite obvious that low salaries prevent a high standard of living among any group. After investigation we conclude that the limited professional training of Negro teachers results from five major handicaps:—poor facilities for training within the financial means of teacher prospects; too low stand-

* Paper read at the Baton Rouge meeting of the National Council of Teachers of Mathematics in December, 1940.

ards required for certification; too large a dependence on experience rather than on training; the overcrowding of the teaching field by Negroes because of lack of opportunity in other professions; and the lack of adequate supervision.

The teachers must depend upon their own experiences for teaching the required reading, writing, arithmetic, and so-called industrial work. How can there be any hope of inculcating within the pupil an understanding of the common, every day mathemaical problems—buying and selling, or taxes, or social security, or saving with the added value of interest and compound interest, if the teacher has had no experience with these practical phases of living?

The problem of the short school term in schools for Negroes like that of professional training for the teachers is full of paradoxes—a well-known one being that there is no need for longer terms for Negro children because they do not attend with regularity. One truth is that the Negro child would attend with greater regularity if he had better housing for extremely bad weather. A comparative statement of the length of the school terms for whites and Negroes in the 17 southern states and the District of Columbia where separate schools exist by law shows a variation for Negroes from 119 days in Mississippi to 187 days in Missouri and for whites from 144 in Alabama to 188 in Maryland.[1] The greatest difference in any one state was in Louisiana where the average length of term in schools for whites was 175 days and 128 in schools for Negroes. There were 553,291 Negro children out of school each day in 1935–36. This was more than one-fifth of the Negro enrollment.[2]

Some indication of the causes of the absence of Negro children from school, especially in rural areas, is given in a study of 34,056 such children in rural communities in 1934. According to this study, "helping at home," "illness," and "working" were the predominant reasons for absence. To these might be added the lack of effective compulsory attendance laws as being responsible for poor school attendance of Negro children.

The influence of the school or lack of it also affects school attendance, and may be the result of poor teaching or poor equipment. Major consideration, however, should be given to the lack of motivation for the Negro child. A strong stimulus for mastery is the pupil's discovery of a personal meaning, an "alive" interest in the subject. The establishment of stimuli by basing the teaching of mathematics on his daily experiences and future plans will go far toward removing absences because of distance from school, minor illness, or other reasons; it will give impetus to the discovery of a personal meaning in his study of mathematics by showing him how an understanding of simple arithmetic may give him economic independence and make him a more intelligent consumer, a better worker, and a more desirable citizen.

Consideration of the problem of the physical equipment of schools for Negroes in this paper is important because physical equipment effects the educational achievement of the child and determines his educational activities.

Data are easily available for the study of the inadequacy of school buildings for Negroes and the almost complete non-provision of other school equipment. Use of free text-books, libraries, and laboratory facilities, and suitable seating are reflected in the higher achievement of school pupils, conversely, low achievement is reflected when these facilities are absent.

A discussion of the inequitable allotment of educational funds will have no place here. Nevertheless attention is called to the low salaries paid to Negro teachers because of such unfair division. The Committee on Equal Opportunity of the Na-

[1] Bulletin 1938, No. 13: Statistics of the Education of Negroes, 1933–34 and 1935–36, United States Department of Interior; Office of Education, p. 35.

[2] *Ibid.*, p. 9.

tional Education Association comments:[3]

"Differences in the level of training and in living costs may account in part for the difference in salaries paid. But there are other influences, deeply rooted in custom and tradition, which also cause inequalities in salary as between racial groups. This practice is not confined to school systems; it commonly exists in all types of gainful employment."

The committee in its summarization states: "Levels of professional training of Negro teachers tend to be lower than those of white teachers. The differences, however, do not account in full for the differences in salary."

Statistics are also available showing the comparison between pupil-teacher ratios for both races. The Negro teacher generally has a higher teaching load than the white teacher in each of the 17 states and the District of Columbia, previously referred to. Sometimes the difference is as great as 15 or 20 pupils, as in Mississippi with 50 for Negroes and 35 for whites, and Louisiana with 49 for Negroes and 29 for whites. Many handicaps result, however, even when the difference is only 2 or 3 as in the District of Columbia.

The District of Columbia is cited because it is considered the only strictly dual system of public school education and hence offers the nearest to equality of opportunity for Negroes in any segregated system. School terms and salaries are equal for both races. Physical equipment is supposed to be on an equitable basis. There is autonomy in instruction and supervision by Negroes on all levels and in all departments. Yet, in the past four years the teaching load has become heavier for Negro teachers than for white teachers.

The situation in mathematics is very bad in our junior and senior high schools. Some teachers of mathematics in the junior high schools have classes of over 60 pupils. Because of the heavy load in the senior high school, another bad effect exists. For the first time in a number of years, there has been no class in trigonometry scheduled in the Negro high schools. This is a definite handicap to those pupils who want the course, some of whom need it to enter the college of their choice. Especially is this bad for those pupils who desire to major in mathematics. While the very idea of warfare is lamentable, yet we are in a crisis when all attention is on defense training. The theme for the Council program at its February meeting is: "Mathematics in a Defense Program." In our high schools of the District of Columbia we place considerable emphasis upon military cadet training. One has only to look at bulletins issued by the War Department on operation of certain types of guns and one could not read those bulletins intelligently without knowledge of trigonometry.

If we multiply the problems in the Washington schools by eight or ten we get some idea as to how the heavy teaching load affects opportunities for Negro children in many other communities.

Many of the problems previously mentioned may be solved through the interest and help of the communities in which they occur. As stated before, it is the business of the teacher to lead the pupils into the discovery of a personal meaning in the teaching of mathematics; it is the business of the community to provide a laboratory for the pupil's experimentation for his growth.

Let us remember how dull would be the mastery of the income tax procedure if there were no need for such knowledge and, some of us, in spite of its need, never master its intricacies. Then think of the bewilderment of the adolescent who is brought into the intricacies of interest, measures, banking, or even simple arithmetical computation unless it is made a part of his own life. How much simpler the problem of standards and measures if the child can see protection for himself as a purchaser. How much simpler if the in-

[3] National Education Association Committee on Equal Opportunity, *Progress and Problems in Equal Pay for Equal Work.* June 1939, p. 29.

terest problem can be taught through understanding the value of a small bank account.

The abstract thought of owning a home may mean little to a child but when it is used as the means of teaching interest, bank problems, insurance, loans, computation of land measures and lumber standards for building a home, the problems become personal. Thus all ordinary topics of mathematics may function in the life of the child if properly taught.

It is in the personalized problems that the facilities of a community become important. There is no limit placed upon the use a white child may make of his community as a laboratory. If it is banking interest, there are local banks that are willing to cooperate; for other problems, utilities lend their facilities, as do business men and the city government itself. It is easy to understand this, for, from the ranks of these boys and girls are recruited their clerks, their managers, and their investors.

Although this paper will make no evaluation of the social and economical problems of the Negro, nor offer any solution for them, it is necessary to call attention to the fact that it is only in rare instances that the community facilities are made available for the Negro child. However, in those sections where Negro business has been well developed, the mathematics of business may become a part of the life experience of the Negro child, for once the pupil acquires a personal interest in a subject, he becomes "alive." He articulates and develops a curiosity for what has before been a boring subject.

It is possible however, for the Negro pupil to be guided into a "mathematics consciousness" through the daily use of consumer problems, work problems, and social security problems of the family of which he is a part. The child learns, the family economic status is raised, and the community has a group of better citizens.

Such laboratory teaching units are frequently a part of school curricula. The Negro pupil, however, does not benefit from his teaching because either his school lacks facilities for such teaching or he has biased psychological reactions to the organization of the community in which he lives but of which he is never really a part.

Particular attention should be called to the policy of subtly handicapping Negro teachers by substituting meaningless industrial programs for necessary foundation subjects without considering the pupil's interest in or aptitude for such work.

These are a few of our problems in the solution of which the National Council and the Negro teacher may serve, both as contributors and participants.

How the Council May Serve Negro Teachers

1. The Council is serving the Negro teachers now by sponsoring an open, frank discussion of these problems on the basis that any school procedure that affects one-tenth of the population of the United States in an adverse manner will eventually affect the remainder of the population in some adverse way. And unless eliminated or placed under complete control, these cancerous situations become progressively disrupting.

2. The Council may serve the Negro teacher of mathematics by directing attention to possible solutions of general phases of teaching problems through cooperative effort. It may use its facilities, including publications, for gathering data, interpreting its findings and publicizing the need for a socialized corrective program for teaching mathematics in schools for Negro children.

The Council and its publications may serve the Negro teachers by calling their attention to the experiences of teachers who have succeeded in giving a personalized meaning to the teaching of mathematics.

3. Although the emphasis in this paper has been on the unfavorable conditions affecting Negro teachers, it must not be for-

gotten that there are many highly trained Negro teachers, many who have superior qualifications to many of the whites in the same localities. This is readily seen when we consider that many of our most intelligent, especially among the women, are found in the teaching profession, because so few other fields have been open to them. Therefore, those school systems which have high requirements and competitive examinations, and offer good salaries have caused many candidates for teaching positions to acquire more than the necessary requirements because of keen competition and realization that only the highest ranking would have chance for appointment. Many of the highly trained ones, though not fortunate enough to receive a position with a good salary have accepted positions in smaller communities and have been contributing much to those communities. The Council might obtain much valuable material from those teachers. The Council will make an excellent beginning in such work by recommending the evaluation of mathematics programs in selected secondary schools with accredited curricula and full terms. Such schools which are completely manned by Negro teachers are found in Washington, Baltimore, St. Louis, Indianapolis, Nashville, Memphis, Atlanta, New Orleans and many other places.

If such an evaluation program is undertaken by the proper educational authorities it will be possible to compare the best features of work in these schools with accepted standards throughout the country with the view to publicizing these comparisons. Pointing out the commendable features of these schools with reference to mathematics, in the opportunities for various types of programs, facilities, teacher training, and pupil progress, will serve as an incentive to other Negro schools to aspire to similar goals, and criticizing any unfavorable situations will serve as a stimulus to the authorities in charge of the schools to remedy the shortcomings.

4. The Council may further serve the Negro teacher of mathematics by a more extensive plan for selling itself to teachers through membership campaigns and through the use of its publications. Here these questions might be asked:

How many state representatives of the Council have been interested in problems affecting schools for Negroes? How many state representatives of the Council have made efforts to interest Negro teachers in the Council? When it is remembered that there are approximately 2300 Negro High Schools in the United States offering at least one year of mathematics and approximately 120 four year colleges, junior colleges and teacher training institutions for Negroes, the field for Council expansion becomes enlarged.

5. It is suggested that the Council through its state representatives and publications advocate for Negro children a program of mathematics to include participation in individual and group activities based upon their individual and group experiences in the community in which they live. Such a program may be sponsored with sincerity because of the social significance of guidance toward better living through study. Any such plan, however, will be useless unless it has as one of its basic principles the fuller integration of the Negro child into the civic life of the community in which he lives at present.

How the Negro Teacher May Serve the Council

Negro teachers usually hear of the Council only through attendance at meetings where such meetings are held in their own city or section, through attendance at large universities where some of their professors are "Council conscious," through reading the publications, or through the state representatives. Thus, only a very small per cent have heard of the Council. (1) More Negro teachers of mathematics in 2300 high schools and 120 Negro Colleges and teacher training institutions should join the National Council. They

may make marked contributions to the development of "Council consciousness" through representation of mathematics teachers in Council activities. These representatives may carry back to their own communities ideas for local organizations of teachers of mathematics. (2) Group organization as such may have lost some of its value but there is no better procedure for arousing interest among Negro teachers in any teaching problem, in this case, mathematics consciousness in all types of teaching. In the eighteen states which have separate schoos for Negroes on both elementary and secondary levels, there are separate Negro educational associations, many of which require attendance at the annual association meetings. The writer has made an unsuccessful attempt to ascertain how many of these associations have organized mathematics sections to substantiate an opinion that many if not all of these state organizations have no organized mathematics sections. The writer was present at the annual meeting of the Kentucky Negro Education Association in 1936 where over 1100 teachers were in attendance. It was noted that there was a section of English teachers, a section of Science teachers, and other sections but no section of mathematics teachers. It seems quite possible to suggest that each of the state Negro teachers associations establish a section of mathematics teachers which will be concerned with the teaching problems in mathematics.

It may be further suggested that each mathematics section so organized assume the financial responsibility of sending one teacher as a delegate to one of the National Council meetings each year.

(3) More teachers must become members of the Council if for no other reason than to read regularly its publications. The mere scanning of publications offers no help in solving problems unless this browsing leads to habitual reading. The use of material in the publications as guides will prove useful. Once a teacher begins to read THE MATHEMATICS TEACHER regularly, she will be surprised to find how helpful it is to her.

(4) Professors of the teaching of mathematics in colleges for Negroes should not only be members of the only organization devoted exclusively to the interests of mathematics teachers, but they should lose no opportunity to acquaint the prospective mathematics teacher with the importance of belonging to the mathematics Council and of the value of THE MATHEMATICS TEACHER, its official organ.

From the writer's experience as an examiner of papers of candidates for teaching positions in mathematics, there have been some candidates with masters degrees from first rate colleges and universities who have not heard of THE MATHEMATICS TEACHER or the National Council of Teachers of Mathematics.

Recapitulating:

The general recommendations are submitted as the basis for securing better results for teaching generally and for mathematics particularly:

1. That, the Council interest itself in and lend support to efforts to better educational backgrounds for Negro teachers through
   - a. Higher professional requirements
   - b. Longer school terms
   - c. More equitable salaries
   - d. Decrease in teaching loads
   - e. Adequate physical equipment.

2. That the Council use its facilities, including publications, for gathering and interpreting data and disseminating knowledge so gained of the work and needs of Negro teachers of mathematics in schools for Negroes.

3. That the Council request from the proper educational authorities an evaluation of a group of selected schools for Negroes manned by Negro teachers.

4. That the Council make efforts to secure representative Negro membership through the cooperation of its state representatives and other desirable media.

5. That the teachers see that mathe-

matics sections are organized in their state teachers associations.

6. That all states send delegates to Council meetings.

7. That teachers develop the habit of reading THE MATHEMATICS TEACHER regularly.

8. That professors in colleges and teacher training institutions for Negroes call attention to the work of the Council.

If these general recommendations are accepted by the Council, a more specific recommendation is herewith submitted: That the Council select a committee composed of members of both races; the committee to be instructed to investigate the feasibility of carrying out some plan as referred to above and that the results of such investigation be presented at some future meeting of the council for action.

An Investigation of Mathematics Teachers in Minnesota

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Any attempt to improve teacher training programs must proceed from a knowledge of conditions under which teachers work. The purpose of the present investigation is to present such information relative to mathematics teachers in Minnesota. In addition to the value of the facts presented, it is hoped, that the investigation will serve as a pattern for similar reports from other localities, so that a more adequate basis for the work of improving teacher training programs will become available.

The present study reveals wide variations not only in the training of mathematics teachers but also in experience, the number and kinds of classes taught, and the enrollment in mathematics classes in the various grades of the high school. Differences in practice are also revealed in different types of school organizations (Table I).

Table I
The Number of Schools and Teachers in Each of the Four Types of Schools

Schools	Number	Men	Women	Total
Junior-Senior High	116	193	223	416
Four-Year High	97	86	46	132
Six-Year High	250	323	125	448
High school Dept.	11	9	2	11
Total	474	611	396	1007

There are 1007 mathematics teachers in the high schools in Minnesota.[1] Six hundred eleven of these teachers are men and 396 are women. They represent 474 schools.

Early in the progress of this investigation it became apparent that one who wishes to teach mathematics in the high schools of Minnesota must be prepared to teach a wide variety of subjects. This is illustrated in the instance of one teacher who taught 162 pupils daily in seven different subjects; mathematics, history, music, biology, chemistry, physical education, and economic geography. This teacher had earned a major in mathematics and a minor in German and had been granted a B.E. degree upon graduation from College. During his first two years of teaching he received a yearly salary of $990, and had earned two quarter credits in graduate work. Like information is available for each of the 1007 mathematics teachers but for the purpose of this paper other means of presentation will be used. There are four types of high school organizations each presenting a different picture of the mathematics teacher. Information is provided for each type in the following pages.

Types of High School Organizations

The 474 schools represented in this investigation, are divided into four types, the Junior-Senior High School, Four-Year, Six-Year, and the High School Department,[2] (see Table I). The Junior-Senior organization is most commonly found in the larger communities. The median and mean number of teachers are 20 and 27, respectively, with a standard deviation of 16.8. The types of communities in which the Four-Year and Six-Year High Schools are found do not differ greatly in size. However, as a group the smaller communities are represented by the Six-Year organization. This can be seen in a comparison of the accompanying measures. The median, and mean number of teachers in the Six-Year School are 7.8 and 7.7

[1] The mathematics teachers of Minneapolis, St. Paul, and Duluth are not included in this investigation.

[2] Minnesota Educational Directory, Department of Education, State of Minnesota, 1938–39. Pp. 19–52.

respectively, with a standard deviation of but 2.4, while the median and mean number in the Four-Year Schools are 4.7 and 7.1, respectively, with a larger standard deviation of 5.8. No Six-Year organization was found in schools having more than fifteen teachers while there are eight such schools of the Four-Year type. The High School Department represents the smallest communities. Eleven schools of this type ranged from two to four teacher schools (Table II).

TABLE II
The Median and Mean Number of Teachers per School in Each of the Four Types of Schools

Schools	Median	Mean	Standard Deviation
Junior-Senior High	20.8	27.0	16.8
Four-Year High	4.7	7.1	5.8
Six-Year High	7.8	7.7	2.4
High School Dept.	3.0	3.0	0.0

THE PROVISIONS FOR MATHEMATICS INSTRUCTION IN THE HIGH SCHOOLS

The mathematics program of a school appears to vary somewhat with the size of the school. Schools of all types provide mathematics in the ninth grade. However, practice varies considerably in the provision made for mathematics in grades ten, eleven, and twelve. The information in Table III shows a marked decrease from grade ten through twelve in the number of schools offering mathematics courses. The decrease depends in a measure upon the size of the school. The largest type of school makes the greatest provision for courses. One hundred five of these schools or approximately 90% offer tenth grade mathematics courses, 61% eleventh grade courses, and 28% twelfth grade courses. The smaller type, the Six-Year High School, shows a sharper decrease in the number of courses offered. Sixty-seven per cent of these schools offer courses of mathematics in the tenth grade, 29% offer courses in the eleventh grade, and but 1½% offer courses in the twelfth grade. As a group 76% of the high schools in Minnesota provide instruction in mathematics in the tenth grade, 37% of them provide it in the eleventh grade, and 8% of them in the twelfth grade.

TABLE III
Number of Schools of Each Type Which Offer Courses in Mathematics in Grades Ten, Eleven, and Twelve

Schools	Grades		
	10	11	12
Junior-Senior High	105	72	28
Four-Year High	81	33	6
Six-Year High	168	72	3
High School Dept.	7	2	2
Total	361	179	39

NUMBER OF PUPILS ENROLLED IN MATHEMATICS COURSES

The enrollment in mathematics classes is reflected by the decrease in the number of schools offering the subject from grades nine through twelve. Twenty-three thousand seven hundred three pupils are enrolled in the ninth grade mathematics class, 11,206 in grade ten, 3275 in grade eleven, and 654 in grade twelve. Of the 654 pupils enrolled in the twelfth grade mathematics class 514 of these are in the 116 Junior-Senior High Schools leaving a

TABLE IV
Number of Pupils Enrolled in Mathematics Classes in the Four Types of Schools

Schools	Grades					
	7	8	9	10	11	12
Junior-Senior High	7,976	8,922	11,903	5,846	1,565	514
Four-Year High	173	185	3,378	1,904	585	74
Six-Year High	4,773	5,049	8,313	3,360	1,113	56
High School Dept.	50	38	109	96	12	10
Total	12,972	14,194	23,703	11,206	3,275	654

balance of but 140 pupils enrolled in the classes of the remaining 368 high schools (Table IV).

Types of Schools in Which the Teachers Were Trained

An investigation of the training of the mathematics teachers reveals that 443 or approximately 44% received their training in the Liberal Arts Colleges, 313 or 32% in Teachers Colleges, and 236 or 23% were trained in State Universities (Table V).

Table V

Types of Schools in Which the Mathematics Teachers Received Training and the Number of Teachers Trained in Each Type

Schools	Teachers College	Liberal Arts College	State University
Junior-Senior High	133	137	139
Four-Year High	27	79	26
Six-Year High	147	225	69
High School Dept.	6	2	2
Total	313	443	236

The B.A. degree, followed closely by the B.E. and B.S. degrees, predominates in the different degrees held by mathematics teachers (Table VI).

subject as a minor teaching field. One hundred twenty-one or approximately thirty-six per cent of the teachers majored or minored in fields other than mathematics.

One hundred thirty-two teachers of the Four-Year High Schools represented 53 different major and minor teaching field combinations. The combination most frequently reported was science-major and mathematics-minor. Twelve of the 132 teachers reported this combination. Mathematics-major and science-minor, and mathematics-major and history-minor were each reported by five teachers. The remaining thirty-one combinations were reported in each case by from one to four teachers.

Thirty-five of the 132 teachers reported a major in mathematics and 39 reported the subject as a minor. Fifty-eight or approximately 43% of the teachers of mathematics in the Four-Year High Schools report that they majored or minored in fields other than mathematics.

In the case of 400 teachers in the Six-Year High Schools 129 different major and minor combinations were reported. The

Table VI

Kinds of Degrees Granted to Mathematics Teachers

Schools	Degrees							
	B.S.	B.A.	B.E.	M.A.	M.S.	Ph.B.	Ph.M.	Ph.D.
Junior-Senior High	99	126	84	31	5	3		
Four-Year High	25	71	23	5	2	1		1
Six-Year High	87	195	108	15	8	3	2	
High School Dept.	2	2	6					
Total	213	394	221	51	15	7	2	1

Major and Minor Teaching Fields of the Teachers

Three hundred twenty-nine teachers in the Junior-Senior High Schools reported 101 different major and minor combinations. The highest frequency in which any combination was reported was for a mathematics-major and a science-minor, nineteen teachers reported this combination. One hundred forty-five reported a major in mathematics and sixty-three reported the

combination most frequently reported was mathematics-major, science-minor which was reported by twenty-six teachers. One hundred thirty teachers reported mathematics as their major teaching field and eighty-three reported mathematics as a minor field. One hundred eighty-seven or approximately forty-six per cent of those teaching mathematics in the Six-Year High Schools have majors and minors in teaching fields other than mathematics.

TABLE VII

Combinations of Major and Minor Teaching Fields of Mathematics Teachers Employed in Junior-Senior High Schools in Minnesota

Major Training	Number Majors	Combinations with Major Fields Listed in First Column Arranged in Order of Frequency
Column 1	Column 2	Column 3
1. Education	31	3; 6; 4; 7; 9; 15; 21
2. Agriculture	1	16
3. Mathematics	145	14; 6; 5; (4-16); 7; 19; (1-8-13-15); 9; (12-17); 10; 11; 21
4. Soc. Science	12	(3-5); (1-6-7-13)
5. English	22	4; 3; 6; (14-15); (10-12-13)
6. History	17	3; 4; 8; 5; (1-13)
7. Chemistry	22	3; 5; (12-14-16); (10-15-19)
8. French	2	(3-5)
9. German		
10. Latin	8	3; (5-6-7-13-14)
11. Ind. Arts	6	4; 3; 14
12. Economics	9	(3-16); (1-4-5-19)
13. Music	2	(3-5)
14. Science	29	3; 5; 6; (4-19); (7-8-9-14-15-21)
15. Biology	10	8; (1-3-5-9-14-19-4)
16. Physics	8	3; 7; (15-19-21)
17. Spanish		
18. Swedish		
19. Physical Ed.	4	(3-4-5-6)
20. Home Econ.	1	14
21. Psychology		

In Table VII are listed the subjects that were combined with the various majors. In the first column of this table are listed the majors; in the second column the frequency with which the majors occurred; and in the third column, the subjects combined with the various majors. The numbers in the third column represent the subjects listed in the first column. The subjects in the third column are listed according to dominance; that is, the subject that occurred most frequently is listed first, the

TABLE VIII

Combinations of Major and Minor Teaching Fields of Mathematics Teachers Employed in Four-Year High Schools in Minnesota

Major Training	Number Majors	Combinations with Major Fields Listed in First Column Arranged in Order of Frequency
Column 1	Column 2	Column 3
1. Education	6	(3-4); (6-16)
2. Agriculture		
3. Mathematics	35	6; 16; (5-14); (4-10); 8; (1-7)
4. Soc. Science	5	(1-3-5-15-19)
5. English	6	(3-6); (4-10)
6. History	8	3; 5; (9-14-16)
7. Chemistry	17	3; 1; (14-15-16); (6-12-13)
8. French	2	3
9. German	3	3; 14
10. Latin	6	3; 7
11. Ind. Arts	2	3
12. Economics	2	(3-6)
13. Music	1	3
14. Science	18	3; 1; (5-7-8-15)
15. Biology	4	5; 7; 14
16. Physics		
17. Spanish		
18. Swedish		
19. Physical Ed		

TABLE IX

Combinations of Major and Minor Teaching Fields of Mathematics Teachers Employed in Six-Year High Schools in Minnesota

Major Training	Number Majors	Combinations with Major Fields Listed in First Column Arranged in Order of Frequency
Column 1	Column 2	Column 3
1. Education	22	3; (6-7-16); (1-10-14-15-17-19-20)
2. Agriculture	2	(1-14)
3. Mathematics	130	14; 5; 6; 16; 4; 19; (7-8); 12; (9-13); 1; 10; 15
4. Soc. Science	15	3; 6; (9-11-13-14-15-19)
5. English	34	3; 6; 8; (4-10-13); (7-9-15); (14-19)
6. History	36	3; 4; 5; (7-12-14); (1-10-11-13-15-19)
7. Chemistry	43	3; 9; 16; (14-15); (1-5-6-8); (13-19)
8. French	2	1; 5
9. German	4	13; (12-5)
10. Latin	5	(1-3-5-9-14-21)
11. Ind. Arts	14	3; (4-5); (6-15)
12. Economics	8	3; 5; (4-6-13); 22
13. Music	4	3; (1-17)
14. Science	47	3; 4; 5; (1-6-9); (13-14-18-19-20)
15. Biology	21	(1-6-7); (3-8); (4-5-9-10-13-14-16-19)
16. Physics	7	3; (4-5-14-18)
17. Spanish		
18. Swedish		
19. Physical Ed.	4	4; (3-15)
20. Psychology		
21. Greek		
22. Norse		

one that occurred with the next greatest frequency is listed second, etc. The parentheses enclosing a group of numbers indicate that the subjects listed occurred with equal frequency. Example: Education is listed as the first major. There were thirty-one teachers who had this major. The 3, 6, 4, 7, 9, 15, and 21 in the third column indicated that the subjects combined with education as minors were mathematics, history, social science, chemistry, German, biology, and psychology. Tables VIII and IX for the Four-Year and Six-Year High Schools are read in the same manner.

Of the 861 teachers reporting major and minor teaching fields, 311 reported mathematics as a major teaching field and 190 reported mathematics as a minor field. Three hundred sixty or approximately forty-one per cent of the teachers of mathematics in Minnesota reported that they majored and minored in fields other than mathematics (Table X).

TABLE X

Number of Different Major-Minor Combinations in Which the Mathematics Teachers of Minnesota Were Trained

Schools	Number of Teachers	Number of Major-Minor Combinations
Junior-Senior High	329	101
Four-Year High	132	53
Six-Year High	400	129
High School Dept.	8	7
Total	861	

TABLE XI

Years of Experience of Teachers in the Four Types of Schools

Schools	Number Schools	Number Teachers	Median Years	Mean Years	Standard Deviation
Junior-Senior High	116	416	13.7	15.1	9.18
Four-Year High	97	132	8	9.7	8.2
Six-Year High	250	448	5.6	7.8	7.2
High School Dept.	11	11	4.2	6.9	4.4
Total	474	1007	9.2	11	7.09

TABLE XII

Quarter Credits of Graduate Work Earned by Teachers in the Four Types of Schools

Schools	Number Teachers	Median Qu. Credits	Mean Qu. Credits	Standard Deviation
Junior-Senior High	225	13.07	18.6	16.6
Four-Year High	82	15.5	18.22	11.92
Six-Year High	220	12.6	17.64	14.2
High School Dept.	Omitted			
Total	537	13.2	18.5	14.04

TEACHING EXPERIENCE OF MATHEMATICS TEACHERS

With respect to teaching experience it appears that those of greatest experience are found in the largest schools. Teachers in the Junior-Senior High Schools show an average of 15.1 years of experience with a standard deviation of 9.18 years. Teachers in the Four-Year High Schools averaged 9.7 years of experience with a standard deviation of 7.2, and those in the High School Department average 6.9 years with a standard deviation of 4.4 years (Table XI).

GRADUATE WORK DONE BY THE TEACHERS

Five hundred thirty-two or approximately 52% of the teachers report they have taken graduate work. They earned an average of 18.5 quarter credits with a standard deviation of 14. Fifty-two per cent of the teachers in the Junior-Senior High Schools reported they had earned credits in graduate work. Sixty-two per cent of the teachers in the Four-Year High Schools had earned graduate credit, as had 49% of the Six-Year High Schools teachers (Table XII).

NUMBER OF CLASSES TAUGHT DAILY BY EACH TEACHER

The number of classes taught daily by the teachers does not vary greatly between the different types of high schools. The 1007 teachers reported that they taught an average of 5 classes a day with a standard deviation of 1.1 (Table XIII).

NUMBER OF MATHEMATICS CLASSES TAUGHT DAILY BY EACH TEACHER

In the case of classes in mathematics taught each day there is a noticeable differ-

TABLE XIII

Number of Classes Taught Daily by Teachers in the Four Types of Schools

Schools	Teachers	Median	Mean	Standard Deviation
Junior-Senior High	416	5.1	4.8	1.24
Four-Year High	132	5	4.7	1.18
Six-Year High	448	4.9	4.7	1.33
High School Dept.	11	6	6	
Total	1007	5.04	4.8	1.17

TABLE XIV

Number of Mathematics Classes Taught Daily by Teachers in the Four Types of Schools

Schools	Teachers	Median	Mean	Standard Deviation
Junior-Senior High	416	3.8	3.7	1.46
Four-Year High	132	2.5	2.7	1.16
Six-Year High	448	2.7	2.9	1.22
High School Dept.	11	2	2	
Total	1007	3.02	3.2	1.38

ence in practice for the different types of schools. It appears that the larger the school the greater the number of mathematics classes in the teacher's daily schedule. Of the 1007 teachers the average number of mathematics classes taught each day is 3 with a standard deviation of 1.3 (Table XIV).

Salaries of Mathematics Teachers

Salaries of mathematics teachers vary widely between the different types of schools. The wide variation between teachers is due somewhat to the fact that mathematics, particularily in the smaller schools, is often taught by the principal or superintendent who are paid a higher salary because of the administrative duties involved in their work. The wide variation between schools as regards to salaries appears to be related somewhat to the variation between schools as to size (Table XV).

Table XV

Annual Salaries of Mathematics Teachers in the Four Types of Schools

Schools	Number Teachers	Median Salary	Mean Salary	Standard Deviation
Junior-Senior High	414	1470	1453	363
Four-Year High	129	1213	1327	350
Six-Year High	422	1129	1283	405
High School Dept.	Omitted			
Total	965	1255	1359	404

Summary

1. The Junior-Senior High School organization represents the largest school in Minnesota, following in order of size are the Four-Year High School, the Six-Year High School, and the High School Department.

2. Sixty per cent of the mathematics teachers in Minnesota are men.

3. Seventy-six per cent of the high schools in Minnesota offer a course of mathematics in the tenth grade, 37% in the eleventh grade and 8% in the twelfth grade.

4. Enrollment in mathematics courses decreases from grade to grade in the high school. Twenty-three thousand seven hundred three pupils are enrolled in ninth grade courses, 11,206 in the tenth grade courses, 3275 in the eleventh grade courses, and 654 in the twelfth courses.

5. Mathematics teachers are required to teach a wider variety of classes in the smaller schools, as well as a larger number of classes.

6. Thirty-six per cent of the mathematics teachers in Minnesota reported that they had majored in the subject in College. Twenty-two per cent minored in the subject. Forty-one per cent reported they had neither majored nor minored in mathematics in College.

7. Forty-three per cent of the mathematics teachers investigated are graduates of private Liberal Arts Colleges, 31% are graduates of State Teachers' Colleges, and 23% are graduates of State Universities.

8. Forty-four per cent of the mathematics teachers in the Junior-Senior High Schools, 27% in the Four-Year High Schools, and 32% in the Six-Year High Schools reported they had earned a major in mathematics in College.

9. Nineteen per cent of the mathematics teachers in the Junior-Senior High-Schools, 33% in the Four-Year High Schools, and 20% in the Six-Year High Schools reported they had earned a minor in mathematics in College.

10. There is a wide variety of major-minor combinations in the college work of mathematics teachers. Fifty-three different combinations were found in the training of 132 teachers in the Four-Year High Schools, 129 different combinations among the 400 teachers of the Six-Year

High Schools, and 101 different combinations among 329 teachers in the Junior-Senior High Schools.

11. It appears that teachers of least experience are found in the smallest schools.

12. Fifty-two per cent of the mathematics teachers in Minnesota reported that they had earned credits in graduate work. They earned an average of 18.5 quarter credits with a standard deviation of 14.

13. Salaries differ widely between schools. It appears that the salaries are related with the size of the school. The largest type of school, the Junior-Senior High School paying the largest salaries, and the smallest type, the Six-Year High School the least.

In the light of these findings it would seem that any program for the improvement of instruction in mathematics addressed to the teachers must take into account the wide range of duties and interests of the teachers.

A survey of instruction in mathematics courses taught throughout the state would be a valuable supplement to the survey of the teachers of these courses.

A Wax Works Show

By James A. Ward
Tennessee Polytechnic Institute, Cookeville, Tennessee

Characters:

A carnival barker
Two stage hands
Five boys dressed as wax figures of Newton, Einstein, Archimedes, and two local mathematicians.
One girl dressed as a wax figure.

Scene:

The inside of a wax works show at a second rate carnival.
The curtain rises to show the barker.

Barker:

Ladies and gentlemen: We are going to present tonight for your approval the greatest exposition on earth. One by one before your very eyes we will bring out a collection of the greatest mathematicians of ancient and modern times that have ever lived on the earth. We will not present their pictures to you, or even their mummified bodies to horrify you. But we will show you wax figures of them, so beautifully and artfully made that you may think they are the genuine people come to life again. But that is not all. These beautiful wax models have been marvelously mechanized by the most ingenious workman so that their limbs are movable and are controlled by intricate clockwork. When they are wound up you can see them performing in a manner wonderful to behold. Ladies and gentlemen, even more marvelous than that, some of them have been wired for sound and you can see them walking, gesticulating and even talking just as they did in real life. The effect is most illuminating, fascinating, colossal, gigantic, cataclysmic, and even the most educational of any performance ever before given. These marvelous wax figures have been one of the art treasures of all the ages preserved with the greatest of care. They have been shown to audiences far and wide. They have been presented before all the crown heads of Europe, and more recently before the bald heads of America.

We will now present our wax figures for your approval. The first is a superb model of a giant of the intellect, the great mathematician, Sir Isaac Newton. He it was who discovered the laws of gravitation and invented Calculus. Calculus is that great branch of mathematics that serves the following three purposes: 1. to help to find areas and volumes, 2. to find speeds without speedometers, and 3. to flunk college sophomores. This great gentleman claims that one day he was sitting under an apple tree (or maybe he was lying) when an apple fell on his head. Thereupon he discovered the law of gravity which reads as follows: Law 73,8291/4: "Article 1. Apples fall down, and not up. Article 2. If you sit under an apple tree long enough, an apple might hit you on the head."

Now some think that Newton's intellect had been strained by the invention of the calculus and that is why he had no better sense than sit under an apple tree. But the later researches show this to be false. Newton did not invent calculus until *after* he was hit, and that is why calculus is such a headache.

Boys, bring out Sir Isaac Newton. We will show him with a model of his famous apple.

(The stage hands now *drag* out Newton and stand him up. He is very stiff, so they bring out a large oil squirt gun and oil all his limbs thoroughly. Then an apple is placed in his hand and one of the stage hands winds him up. (This is accomplished by making a loud grinding noise behind him.) As soon as this is over Newton starts rapidly raising and lowering the apple very

stiffly. He starts out rapidly and soon runs down, making sure to stop in the middle of a motion upward.)

Barker:

That's O.K. boys. Put him back out of the way, and get the next figure.

(He is taken to the background.)

Barker:

The most notorious of all living mathematicians is Albert Einstein. We will present for your approval a wonderful wax figure of this gentleman that looks just like him. Mr. Einstein has invented the wonderful theory of relativity that is supposed to be extremely difficult to understand. But this figure has been wired for sound that you may hear him give his own explanation of the basis of his theory just as in real life. It is really marvelous how simple mathematics is when a great man explains it. Boys, bring out Einstein, and be careful his wig does not fall off. At our exhibition before the German Ambassador last week, someone swiped his wig as a souvenir and our new one hasn't come yet.

(Einstein is now brought out and oiled as was Newton, his mouth is also oiled.)

Barker:

I want you to notice how distinctly he enunciates. Wind him up boys. Now, Einstein, tell us your greatest mathematical discovery.

(Einstein begins in a high voice and speaks rapidly at first and runs down.)

Einstein:

One and one is two. One and one is two. One and one is two. One and one is two. One a-an-nd on-ne i-i-is-s———

Barker:

He has probably forgotten again. Take him to the rear, boys.

There have been no women mathematicians great in the sense that the men are great. They have their own way of achievement. However they have been tremendously important and useful, for we all know that any problem is made much clearer when we have to go with it a beautiful figure. Boys, bring out the figure.

(They bring out a girl in evening dress. She is oiled and wound. Her performance is to very stiffly turn around about once and a half.)

Barker:

This graceful figure illustrates Mae West's famous theorem: "A curved line is the loveliest distance between two points." Put her back with the others, boys.

We will next present a great living Mathematician, who knows a tremendous number of things: Mr. ———— ———— ————. He gets all his extra knowledge from the students he teaches by giving them a quiz every day.

We want you to hear the cheery greeting he gives as he enters his class every morning before that quiz. Mr. ————

(When he is brought out oiled and wound he says the usual greeting of that professor.)

Barker:

The next wax figure we will bring is also a living character, one of the greatest in the country. He is the wisest and most learned of all our dummies. We want to show him in the characteristic pose that won for him the nick-name "Smiling Doc." Bring him out, boys, and make him smile. Ladies and Gentlemen, Dr. ————.

(His performance is to turn his head from side to side and smile. He gradually runs down.)

Barker:

Ladies and Gentlemen, the next great mathematician we are going to produce for you is that of Archimedes, the greatest mathematician of antiquity. He discovered more pure mathematics than did all his predecessors before him. One of his greatest achievements was to prove that a

ball is round. He was one of those scholarly gentlemen who are always thinking of great things. The idea for one of his most stupendous discoveries came to him one day while he was taking a bath. This so excited him that he jumped from the tub and ran down the street naked. . . . Pardon me ladies. Ran down the street nude shouting: "Eureka! Eureka! Eureka!", which is Greek for, "I have found it." The effect on the populace was tremendous. He thought he was wrapped in thought, but they knew better.

Archimedes has the best set of clockworks to make him perform of any of the models you have seen heretofore. More than that, he is wired for sound that you may hear his resounding "Eureka's." We wanted to present the bare facts of this scene as it happened with Archimedes wrapped in thought alone, but that has been censored. In our disappointment we turned to fig leaves, but unfortunately they are out of season. So the best we have been able to do is bathing trunks. Boys, bring out Ark.

(Archimedes' costume consists of bathing trunks and a false beard. Since dragging would be bad on his feet he is brought in feet first. His performance is to pretend to run very fast, but stand in one place and turn around slowly. In the middle of his performance he gives two loud falsetto *Eurekas*. Then is taken to the rear with the others.)

Barker:

Now, Ladies and gentlemen, in order that you may see what a convention of Mathematicians is like, I am going to have my assistants wind these up again in order that you may see them all performing at once. Go ahead, boys.

(They all are wound and perform simultaneously.)

CURTAIN

Note: The success of the show depends on the stiffness and mechanical actions of the wax figures. They keep the same facial expression throughout. The assistants can get many laughs by putting them in ridiculous positions after they have performed. A convenient "shelf" for the oil can is the stiff hand of one of the figures.

Pythagoras

By OLA ESKELSON, *Borger, Texas*

When several hundred years ago
The learned Pythagoras
Sat in his outdoor studio
Before his eager class;

He talked of angles, right and straight,
Diagonals and sides;
Gave council how to calculate
With formulas as guides,

An area or hypotenuse,
A volume or a base:
Geometry's eternal use
For every populace.

A Functional Revision of Plane Geometry

By P. H. Nygaard
North Central High School, Spokane, Washington

I. Should There Be Revision?

Many high school teachers of plane geometry introduce their subject by proudly stating that it is over 2000 years old. They announce in awe inspiring phrases that Euclid wrote his plane geometry textbook before the time of Christ, and that the present course is much as he left it. They insist that revision has been unnecessary because mathematical truths never change. To illustrate their view, they argue along the line that if the sum of the angles of a triangle was 180° in Euclid's day, the sum will be 180° today and forever. This infallibility, or permanence, they insist is one of the inherent characteristics of mathematics, and is sadly lacking in other subjects, especially the sciences.

Such boastfulness is, however, largely misplaced. Mathematics teachers should be thoroughly ashamed of a subject as decrepit as plane geometry. Since Euclid there have occurred countless developments which should have logical repercussions in the teaching of geometry.

One of these developments has been the extensive replacement of deductive reasoning by inductive reasoning. Aristotle and Euclid proved specific truths by showing that they were the consequences of God-given, obvious, general laws. From the time of Roger Bacon and Galileo this procedure has been largely reversed—general laws are derived through a process of classifying numerous specific facts. Nor has this upheaval been confined to science alone; the very theorem just mentioned regarding the sum of the angles of a triangle has been questioned by modern mathematicians. Geometry can ill afford to depend entirely on deductive reasoning when nearly all science and philosophy rely on inductive pragmatism.

A second development which should have had an important effect on plane geometry is the tremendous increase in the applications of mathematics since Euclid's time. To the Greeks plane geometry was the big center of attraction, and was studied as a self-sufficient subject. Conditions now are entirely different. The technical, scientific achievements of modern civilization are a form of living mathematics that cannot be ignored. There is probably as much geometry involved in Coulee Dam or the Golden Gate Bridge as in all the pages of Euclid. This applied geometry should be incorporated as thoroughly as possible into the teaching of the subject.

A third consideration is that plane geometry crystallized before man had invented an efficient number and measuring system. Until the advent of the Hindu notation, the operations of arithmetic and algebra were discouragingly cumbersome. The Greeks also suffered from the lack of uniform units to measure distances and angles. It is, therefore, not surprising that Euclid used line segments to represent numbers, solved proportions by making geometric constructions, talked of angle sums in terms of straight angles instead of degrees, and postulated a straight edge having no units whatever marked on it. Geometry can no longer maintain its place without giving extensive recognition to the effective tool methods of measurement, arithmetic, and algebra.

In our opinion, then, plane geometry is outmoded—first, with respect to its reliance upon a one-sided deductive type of logic; secondly, because of its abstractness, which disregards applications to the works of man; and thirdly, from the standpoint of its poor correlation with the methods of arithmetic and algebra. Our re-organization plan will be directed mainly toward remedying these three deficiencies.

II. What Can Be Omitted?

The writer has made some effort to determine what material, usually included in plane geometry, is of little or no value in later mathematics or science courses. He is convinced that a number of the theorems dealing with the circle have no future use—for instance, the measurement of all sorts of angles in terms of their intercepted arcs. Complicated ruler and compass problems, such as the construction of a triangle having given c, h_c, and m_c, have no conceivable utility in later mathematics. Theorems based on dividing a line segment externally are in the same class. A good house-cleaning along this line will in no way jeopardize the college preparatory value of plane geometry.

Other time consuming items can be eliminated by applying the criterion that it is unwise to thresh over in minute detail ideas which the students have studied in the 8th or 9th grade. This would result in leaving out much of the laborious introduction devoted to definitions and explanations. Comprehensive proofs of the theorems dealing with the area of rectangles, parallelograms, triangles, and trapezoids would come under the same ban.

Still further curtailment can be effected by not giving any time to the formal proof of relationships so simple that the class can see their truth at a glance. It must be discouraging to an enthusiastic beginner in geometry to find that he must apply its high powered technique to the proof of such trivialities as "All straight angles are equal." A far better attitude can be created by proving only the relationships whose truth is open to some reasonable doubt at the outset. Illustrations of "must" theorems are: "The sum of the angles of a triangle is 180°" and "In a right triangle the square of one leg plus the square of the other leg equals the square of the hypotenuse." On the other hand it is gratuitous to prove that alternate interior angles of parallel lines are equal or that equal arcs in a circle have equal central angles. Many of the relationships involved in triangles, parallelograms, and circles could be more efficiently presented as lists of characteristic properties or as student exercises than as theorems completely proved in the textbook. Because the early Greeks were led by their enthusiasm to prove everything in sight, it does not follow that present-day high school students should let loose their formal deductive logic whenever and wherever there is a chance to use it.

III. What Can Be Added?

By omitting from plane geometry all the material suggested in the previous section it will be possible to find place for many valuable additions. Besides stressing deductive reasoning of the hypothesis and conclusion variety, the course can include the use of inductive and experimental methods. It is true that some recent geometry textbooks do a little along this line, but they do so mainly with the intention of showing how weak such methods are. For instance, students are told to measure the angles of a triangle and to find their sum by arithmetic. After doing this for several triangles and always getting approximately 180°, they are asked if it is now proved that the sum of the angles of a triangle is always 180°. Any answer except "no" is frowned upon. In other words the students are led to believe that methods based on measurement and successive trials are too foolish for serious consideration. In our opinion the inductive methods ought to be recognized as being fully as valuable as the deductive.

The use of arithmetic and algebra should receive much attention. The work dealing with angles, proportion, similar triangles, and areas should be thoroughly correlated with real measurement problems solved by numerical methods. Elementary trigonometry should not be relegated to the position of an after-thought at the end of the year. To facilitate algebraic methods, the lengths of segments should be represented by only a single letter instead of by the two letters at the ends of the segment. Authors of modern plane geometry books,

still following the precedent set by Euclid, seem to think that algebraic rules must never be allowed to attain the status of acceptable reasons. They slide from an expression like $a+2a$ to its equivalent $3a$ by the use of the word "or," when it seems that they might as well admit that the reason is the algebraic process of combining like terms. Similarly, changes from x^2-b^2 to $(x-b)(x+b)$ and from $2x/2a$ to x/a are passed over with hypnotic hush, instead of pointing out that the first is based on algebraic factoring and the second on the arithmetic method of reducing fractions to simpler form. Since algebra is being less intensely covered in the 9th grade than was formerly the case, it is important that plane geometry be taught in such a way that algebraic methods are encouraged and developed.

The most obvious direction in which plane geometry must be enlarged still remains. It is toward the immense field of applied mathematics found in the activities that go to make up the complicated, mechanical civilization in which we live. A suggestive list of such possibilities will be given.

The "parallelogram" law should be used in finding the resultant of forces and velocities. These problems involve scale drawing, the hypotenuse rule, and elementary trigonometry. They are of timely importance in connection with airplane navigation.

The interesting topic of "center of gravity" can be entered by way of the theorem dealing with the intersection point of the medians of a triangle. This leads to the stability of automobiles on side hills and in going around curves, the balancing of rotating mechanisms, and a good opportunity for experimental work.

There is much practical geometry involved in the reflection and refraction of light. Congruent and similar triangles, accurate drawings, and circular and parabolic curves are needed in this topic.

In solving problems by means of proportions, it is essential that much practice be given in the use of subscripts and primes. Charles' Law and Boyle's Law might well be utilized for practical problems. Inverse proportions are so often encountered in later science and mathematics that they cannot be completely ignored as is usually done in present geometry textbooks. Such proportions can be introduced by simple geometric illustrations like the relationship between the length and the width of a rectangle having a constant area.

Simple surveying and measurement problems should be worked out by the class. These require some knowledge of measuring instruments, ability to make scale drawings, and an understanding of unavoidable errors of measurement. An elementary study of blueprints would be desirable. The laying out of baseball diamonds and football fields, making scale drawings of house plans, and finding the per cent of grade of streets would provide suitable projects. In this connection students should receive training in the metric system, and in the use of decimals to replace common fractions and radicals.

The study of arcs and tangents has a direct bearing upon railway and highway construction problems. Applications in this field should be included.

Circles and regular polygons furnish the basis for nearly all designs and patterns. Instead of presenting a few pictures of designs to dress up their plane geometry books, the authors should give this topic more thorough attention. This is an application of geometry in which girls might be particularly interested, whereas many of the uses of geometry seem to appeal mostly to boys.

The simple machines of physics, such as the lever, inclined plane, and screw can be used as the source of many problems dealing with proportion, angles, and circles. They could also furnish the groundwork for some experimental verification.

With war all around us, some reference to the military uses of geometry might be appropriate. Mathematicians are, perhaps,

the least warlike of people, but, nevertheless, they are indispensable to the conduct of war. A big gun cannot be aimed unless someone has first figured out angles, distances, and triangles. Navigation of ships at sea and of airplanes in the sky is based on the measurement of angles. Most of these problems are too complex for plane geometry classes, but some simple ones can be included, and even some difficult ones can be handled by the scale drawing method. Some geometric theorems have implications not obvious on the surface. Thus, one would not suspect that the theorem proving that a tangent is the mean proportional between the secant and its external segment would have a military significance, but by its use, together with a little square root, a student can easily find that the horizon for an aviator 4 miles above the ocean's surface is about 180 miles away. Hence this theorem has a direct bearing upon the efficiency with which German planes over the Atlantic are able to spot British ships.

Many other applications of geometry could, no doubt, be included. It is hoped, however, that the items mentioned will serve to indicate the type of revision here advocated.

IV. What Arrangement Should Be Used?

Drastic changes can well be made in the traditional sequence and grouping of topics in plane geometry. The "Book" arrangement of Euclid is not advisable. It is suggested that the topical order be about as follows: congruent triangles, parallel lines, polygons in general, proportion, similar polygons, hypotenuse rule and trigonometry, areas, circles, and regular polygons. The chief innovation in this sequence is that such topics as proportion, similar triangles, and the solution of right triangles, which are the basis for most of the applications of geometry, are introduced earlier in the course than is usually done in order to take full advantage of their usefulness. The first part of the course should be mainly deductive, but there should be gradual inclusion of other methods. Proofs and problems should combine geometry with algebra and arithmetic. Practical applications should be included in each topic so as to correlate closely with the theorems studied. In each topic only a few key theorems should be completely proved in the textbook. Much of each topic should be presented so as to challenge the students to make their own investigations and to report the results of their own research.

Retention of the rigid two column arrangement of the statements and reasons of a proof is not advised. This pattern is found nowhere in advanced mathematics, and would, therefore, have to be unlearned later on. Furthermore, students are likely to follow its form blindly without giving much thought to the main thread of the argument. A more flexible style is preferable. To illustrate, here is a sample of a customary, formal proof:

1. $\angle a = \angle b$.	1. Vertical angles are equal.
2. $\angle b = \angle c$.	2. Given.
3. Therefore, $\angle a = \angle c$.	3. A quantity may be substituted for its equal.

A better form, in our opinion, would be:

Since vertical angles are equal, we know that $\angle a = \angle b$.
It is given that $\angle b = \angle c$.
By substituting $\angle c$ for $\angle b$, it follows that $\angle a = \angle c$.

This second pattern has a further advantage in its close conformity to the style used in ordinary English composition.

V. Will It Be Done?

The desirability of many of the features included in this plan for re-organizing plane geometry may, of course, be open to argument. However, unless some radical plan is adopted for re-vitalizing it, the subject will be hard to defend as part of the high school curriculum.

Inertia always operates to make such reforms difficult to accomplish. Textbook writers and companies are, for financial

reasons, inclined to favor the "status quo." Institutions of higher learning and state boards of education have entrance requirements, examinations, and prescribed outlines of secondary mathematics which penalize change. This is especially true in the eastern part of the United States. However, it is our opinion, that we, teachers of mathematics, have no one to blame but ourselves. It is easy for us to say that our failure to improve geometry is due to lack of suitable textbooks or to the unfriendly attitude of the colleges. We should not hide behind these excuses. By striving individually and collectively to make geometry a functional, living reality we can go about our work with new zeal and confidence.

Christmas Meeting of The National Council

Save time to attend the Christmas meeting of the National Council of Teachers of Mathematics.

The Place: Lehigh University, Bethlehem, Pennsylvania.

The Dates: December 31st, 1941 and January 1st, 1942.

The Program: Two general sessions, sessions for senior high school, junior high school, teacher training, and multisensory aids.

The Hotel Bethlehem offers the following rates:

Rooms with Running Water

Single rooms:	\$2.50 and \$2.75
Double rooms:	\$3.75
Twin bedrooms:	\$4.00

Rooms with Bath

Single rooms:	\$3.00, \$3.50, and \$4.00
Double rooms:	\$4.50, \$5.00, and \$6.00
Twin bedrooms:	\$6.00 and \$7.00

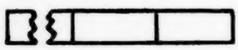
A Presentation of Fractions to Avoid Pitfalls in Arithmetic and Algebra

By Luella E. Leete
Winona, Minnesota

Fractions probably cause more difficulty than any other subject in the mathematics taught in the elementary schools and the junior and senior high schools. Even the bright pupils in the engineering schools have trouble with fractions. A similarity or uniformity of presentation throughout the grades and high schools may help to steer the students away from the pitfalls which are usually found along the "fraction highway." It is with this thought in mind that I offer a few suggestions for the presentation of arithmetic and algebraic fractions.

First, what is the *meaning of a fraction?* In arithmetic a fraction is usually defined as one or more of the equal parts into which a unit has been divided.

One section represents 1/4 of the whole, two sections represent 2/4 etc.; in like manner in



one section represents $1/n$ of the whole, two sections represent $2/n$, while the whole represents n/n. In arithmetic when 1/4, 2/4, etc., are presented, express them also as one-fourth, two-fourths, etc. so that the denominators appear as words—similar to one pencil, two pencils, etc. This will aid in avoiding the pitfall of adding and subtracting 2/3 and 1/2 without changing them to equivalent fractions with the same denominator.

But this arithmetic definition of a fraction cannot be explained with such fractions as 2/3.5 or $(x+2)/(x+y)$. It is then that the algebraic definition of a fraction proves itself to be superior; namely, a fraction is an *indicated quotient.* The dividend and the divisor, now known as the numerator and denominator, can be any kind of a number—integer, fraction, decimal, or polynomial.

Now before a student can simplify fractions or combine fractions by any one of the four fundamental operations he must understand the *fundamental principle of fractions;* namely, what he can do to a fraction to change its appearance but not its value, or how he can change a fraction to an equivalent fraction?

Arithmetic	*Algebra*
1. $\dfrac{5}{8} \neq \dfrac{5+2}{8+2}$ or $\dfrac{7}{10}$	1. $\dfrac{x}{y} \neq \dfrac{x+2}{y+2}$
2. $\dfrac{5}{8} \neq \dfrac{5-2}{8-2}$ or $\dfrac{3}{6}$	2. $\dfrac{x}{y} \neq \dfrac{x-2}{y-2}$
3. $\dfrac{5}{8} = \dfrac{5\times 2}{8\times 2}$ or $\dfrac{10}{16}$	3. $\dfrac{x}{y} = \dfrac{2x}{2y}$
4. $\dfrac{6}{9} = \dfrac{6\div 3}{9\div 3}$ or $\dfrac{2}{3}$	4. $\dfrac{x(x+4)}{x(x+5)} = \dfrac{x+4}{x+5}$

In arithmetic we may have the students find out for themselves if the new fractions formed are equivalent to the original fractions by folding paper into equal sections as we did when we explained the meaning of a fraction. Will the fold for 7/10 coincide with the fold for 5/8, etc.? We, therefore, find that the value of a fraction is not changed when the numerator and denominator have both been multiplied or divided by the same number. But here is the opportunity to stress the difference between *term* and *factor*. Never use the word *cancel* in simplifying fractions, but divide numerator and denominator by all the common factors. If the above form is used in presenting arithmetic fractions, there will be fewer errors in algebra.

Therefore, the *rule for reducing any fraction to lowest term* will be:

1. If not already in factored form, factor the numerator and denominator into prime factors.
2. Divide out of numerator and denominator every *factor* common to both.
3. The result will be the fraction reduced to lowest terms.

Arithmetic:

1. $$\frac{4}{8}=\frac{2\times2}{2\times2\times2} \text{ or } \frac{1}{2}$$

2. $$\frac{48}{100}=\frac{2\times2\times2\times2\times3}{2\times2\times5\times5} \text{ or } \frac{12}{25}$$

Algebra:

1. $$\frac{cx}{dx}=\frac{c}{d}$$

2. $$\frac{ab}{a^2}=\frac{ab}{aa} \text{ or } \frac{b}{a}$$

3. $$\frac{x^2+2xy+y^2}{x^2-y^2}=\frac{(x+y)(x+y)}{(x+y)(x-y)} \text{ or } \frac{(x+y)}{(x-y)}$$

In the final fraction place all polynomials in parentheses, as in (3) above to avoid the common error of dividing a numerator and denominator by a common *term* instead of *factor*. Since factors are multiplied, they appear in parentheses.

Before presenting addition and subtraction of fractions, students should have sufficient drill in changing fractions to equivalent fractions having a certain denominator. Again, a similar form is suggested for arithmetic and algebra:

Arithmetic:

1. $$\frac{3}{4}=\frac{?}{16}$$

2. $$\frac{5}{6}=\frac{?}{24}$$

Algebra:

1. $$\frac{3}{x}=\frac{?}{7xy}$$

2. $$\frac{5}{x+2}=\frac{?}{6(x+2)(x-3)}$$

Procedure:

1. $$\frac{3}{4}=\frac{3\times4}{4\times4} \text{ or } \frac{12}{16}$$

2. $$\frac{5}{6}=\frac{5\times4}{6\times4} \text{ or } \frac{20}{24}.$$

1. $$\frac{3}{x}=\frac{3(7y)}{x(7y)} \text{ or } \frac{21y}{7xy}$$

2. $$\frac{5}{x+2}=\frac{5(6)(x-3)}{(x+2)6(x-3)} \text{ or } \frac{30(x-3)}{6(x+2)(x-3)}$$

After students have had sufficient practice in changing fractions to equivalent fractions either by multiplying or dividing both numerator and denominator by the same factor, they are ready to *add* and *subtract* fractions.

Arithmetic:

1. $$\frac{5}{8}+\frac{3}{16}+\frac{1}{4}=$$

Change each fraction to an equivalent fraction having the L.C.D. for the denominator.

$$\frac{5\times2}{8\times2}+\frac{3}{16}+\frac{1\times4}{4\times4}=$$

$$\frac{10}{16}+\frac{3}{16}+\frac{4}{16}=$$

Combine the fractions into one fraction.

$$\frac{10+3+4}{16}=$$

$$\frac{17}{16} \text{ or } 1\tfrac{1}{16}$$

Algebra:

1. $$\frac{a}{x}+\frac{b}{y}=$$

$$\frac{ay}{xy}+\frac{bx}{yx}=$$

$$\frac{(ay+bx)}{xy}$$

2. $$\frac{a+b}{8}-\frac{a-b}{4}=$$

$$\frac{a+b}{8}-\frac{2(a-b)}{2(4)}=$$

$$\frac{a+b-2(a-b)}{8}=$$

$$\frac{a+b-2a+2b}{8}=$$

$$\frac{(-a+3b)}{8}$$

3. $$\frac{6}{y^2+5y+6}-\frac{y}{y+3}=$$

$$\frac{6}{(y+3)(y+2)}-\frac{y}{y+3}=$$

$$\frac{6}{(y+3)(y+2)}-\frac{y(y+2)}{(y+3)(y+2)}=$$

$$\frac{6-y(y+2)}{(y+3)(y+2)}=$$

$$\frac{(6-y^2-2y)}{(y+3)(y+2)}$$

4. $$\frac{x+2}{x+4}-\frac{x-1}{x+6}=$$

$$\frac{(x+2)(x+6)}{(x+4)(x+6)}-\frac{(x-1)(x+4)}{(x+6)(x+4)}=$$

$$\frac{(x^2+8x+12)-(x^2+3x-4)}{(x+4)(x+6)}$$

$$\frac{x^2+8x+12-x^2-3x+4}{(x+4)(x+6)}=$$

$$\frac{(5x+16)}{(x+4)(x+6)}$$

In both arithmetic and algebra, the following *rule* was observed:

1. Change the given fractions to equivalent fractions having the same denominator (L.C.D.).
2. Add the numerators and place the sum over the common denominator, using parentheses when necessary.
3. Remove the parentheses and write the algebraic sum of the resulting terms over the L.C.D.
4. Combine similar terms in the numerator.
5. Reduce the resulting fraction to lowest terms.

We will find that the steps in the procedure follow those required in our preliminary drill work with individual fractions. Then, too, we will notice that the steps are placed below each other, term by term. In so doing students should avoid two pitfalls—first, that of throwing away of denominators when adding and subtracting fractions after they have cleared of fractions in solving fractional equations; second, that of placing equal signs after both members of an equation. Therefore, nothing should ever appear after an equal sign except the right member of an equation.

Illustrations of *errors to be avoided:*

1. $$\frac{a}{x}+\frac{b}{y}=?$$

$$\frac{a(xy)}{x}+\frac{b(xy)}{y}\neq ay+bx$$

2. $$\frac{x+16}{6}=\frac{x+4}{3}$$

$$\frac{6(x+16)}{6}=\frac{6(x+4)}{3}=x+16=2(x+4)$$

etc.

In *multiplication* of fractions the following rule is suggested for both arithmetic and algebra:

1. Multiply the numerators for a new numerator and the denominators for a new denominator, having each in factored form.
2. Reduce the resulting fraction to lowest terms.

Arithmetic:

1. $\frac{3}{5}\times\frac{10}{9}=$

$\frac{3\times5\times2}{5\times3\times3}=$

$\frac{2}{3}$

Algebra:

1. $\frac{ax}{by}\cdot\frac{cy}{dx}=$

$\frac{a\,cx\,y}{bd\,x\,y}=$

$\frac{ac}{bd}$

2. $\frac{xy}{1}\cdot\frac{x}{y}=$

$\frac{xxy}{y}=$

x^2

3. $\frac{5}{x+2}\cdot\frac{x-5}{10}=$

$\frac{5(x-5)}{5(2)(x+2)}=$

$\frac{(x-5)}{2(x+2)}$

4. $\frac{x^2+8x+16}{x^2-9}\cdot\frac{x-3}{x+4}=$

$\frac{(x+4)(x+4)(x-3)}{(x-3)(x+3)(x+4)}=$

$\frac{(x+4)}{(x+3)}$

If we as teachers stress this rule for multiplication and refuse to introduce or allow "cancellation," the students will not later cancel across any sign, in addition, subtraction, or in equations as—

1. $\frac{2x}{5}+\frac{\overset{2}{10}}{7}=?$

2. $\frac{2x}{5}=\frac{\overset{2}{10}}{7}$

In *division* invert the divisor and multiply, showing each step in the procedure as usual.

Arithmetic

1. $\frac{5}{9}\div\frac{7}{12}=$

$\frac{5}{9}\times\frac{12}{7}=$

$\frac{5\times3\times4}{3\times3\times7}=$

$\frac{20}{21}$

Algebra:

1. $\frac{3x}{5y}\div\frac{6x^2}{7y^2}=$

$\frac{3x}{5y}\cdot\frac{7y^2}{6x^2}=$

$\frac{3(7)x\,y\,y}{5(3)(2)x\,x\,y}=$

$\frac{7y}{10x}$

2. $\frac{x^2-7x+10}{x^2-16}\div\frac{x-2}{x+4}=$

$\frac{x^2-7x+10}{x^2-16}\cdot\frac{x+4}{x-2}=$

$\frac{(x-5)(x-2)(x+4)}{(x-4)(x+4)(x-2)}=$

$\frac{(x-5)}{(x-4)}$

In presenting

$$\frac{3x}{5y}\div\frac{6x^2}{7y^2},\qquad \frac{\overset{7}{21}\;x\;\overset{y}{y^2}}{\underset{10}{30}\;\underset{x}{x^2}\;y}$$

should not be allowed for the third step as then the students will overlook some of the factors which should appear in the final fraction $7y/10x$.

In algebra we have one more topic to present; namely, *fractional equations*, the rule for the solution of which follows:

1. Make every term a fraction by placing 1 under all integral terms.
2. Remove parentheses if necessary by multiplication.
3. Clear the equation of fractions by multiplying each term in both members of the equation by the L.C.D. actually placing the L.C.D. in front of each term.
4. Clear the equation of parentheses.
5. Combine like terms.
6. Solve for the unknown.
7. Check the root in the original equation. But if the check would be more difficult than the solution, resolve the equation to test the accuracy of the root.

Illustrations showing each step of rule:

1.
$$\frac{a}{2}+\frac{a}{3}+\frac{a}{4}=\frac{9}{1}$$
$$\frac{12(a)}{2}+\frac{12(a)}{3}+\frac{12(a)}{4}=\frac{12(9)}{1}$$
$$6a+4a+3a=108$$
$$\frac{13a}{13}=\frac{108}{13}$$
$$a=\frac{108}{13} \text{ or } 8\tfrac{4}{13}$$

2.
$$\frac{y+3}{2}-\frac{3y+11}{4}+\frac{2}{1}=0$$
$$\frac{4(y+3)}{2}-\frac{4(3y+11)}{4}+\frac{4(2)}{1}=4(0)$$
$$2(y+3)-(3y+11)+8=0$$
$$2y+6-3y-11+8=0$$
$$-y-5+8=0$$
$$-y+3=0$$
$$3=y$$

Don't be "left minded" and think that the unknown must be in the left member of the equation.

Check:
$$\frac{y+3}{2}-\frac{3y+11}{4}+2=0$$
$$\frac{3+3}{2}-\frac{9+11}{4}+2=0$$
$$\frac{6}{2}-\frac{20}{4}+2=0$$
$$3-5+2=0$$
$$-2+2=0$$
$$0=0$$

Since these suggestions have proved to be very satisfactory in my algebra classes, I feel that greater progress can be made if teachers of arithmetic and algebra will unite in adopting them to help clear the fraction highway of pitfalls.

Arithmetic

No SINGLE instrument of youthful education has such mighty power, both as regards domestic economy and politics, and in the arts, as the study of arithmetic. Above all, arithmetic stirs up him who is by nature sleepy and dull, and makes him quick to learn, retentive, shrewd; and aided by art divine he makes progress quite beyond his natural powers. *Plato's Laws* 747B. Contributed by Mrs. Pearl L. Weber, Omaha, Neb.

THE ART OF TEACHING

A Device for Teaching Locus

By MARGARET C. AMIG
Washington, D. C.

BY THE time the pupil has reached locus problems (about the beginning of the second semester of plane geometry), he is pretty well acquainted with figures and has learned to observe relationships between their parts—points, lines, and angles. He has been taught to pick the hypothesis out of a statement and draw a figure to illustrate it. With the figure before him, his task is then to discover or to prove certain conclusions about the figure. Locus problems must be approached in a very different way. No description of the figure is presented to the pupil nor is he expected to draw a figure all at once. The problem is to use certain key points or lines in order to locate a set of points which obey certain given conditions. In other words the pupil is to show how a figure *grows* rather than merely to produce one.

Most pupils find this change in point of view from static to dynamic difficult to grasp. Many devices have been suggested to help him, from working models to pin pricks through paper. A plan which I have found very satisfactory, utilizes the bulletin boards with which my classroom is generously supplied. It has advantages over the construction of working models in that it can be used right in the classroom and requires only the simplest and most easily procurable materials—pins or thumbtacks and paper, and demands no initial manual skill.

On a large piece of paper tacked to the bulletin board are drawn the given lines or points. One pupil or if necessary, two pupils acting as partners, are supplied with the necessary equipment—a circle to roll, or a line to move about or a ruler for measuring distances, etc.—and told to carry out the required movements on the paper, using a thumbtack or black-headed pin to mark successive positions of the moving point. The result is a set of points which show the locus clearly as a pattern, traced by the moving point. The illustration shows a parabola marked out with black-headed pins. This is a particularly striking result in that it is entirely unexpected by the pupil. The conditions which the moving point obeys are simple. The pupil can obey them without difficulty, and yet he cannot jump to conclusions before he has a sufficient number of pins to show the existence of a smooth curve. Colored thumbtacks are a little more difficult to handle than the large-headed pins but they add interest and variety to the procedure and make a firmer pattern.

Please Renew May Expirations Now!

MEMBERS of The National Council whose subscriptions expired with the May number of THE MATHEMATICS TEACHER have received notices of the fact together with a request to renew their subscriptions at once. These notices were sent out early to forestall the competition of summer and vacation needs which sometimes cause renewals for THE MATHEMATICS TEACHER to be sidetracked for several months.

Since there are no issues of THE MATHEMATICS TEACHER in June, July, August, and September, subscriptions which started with the October 1940 number closed with the May 1941 number. Those who became members last fall, starting with the October number, are also urgently asked to make their renewals now for the coming year in the light of the considerations outlined above. In order that no one may miss a number the October issue has been sent to all who have not yet renewed their subscriptions.

The National Council Meeting of Teachers of Mathematics
Boston, Massachusetts, June 30, July 1, 1941

By Edith Woolsey, *Acting Secretary*

Another summer meeting of the Council, the seventh convention in cooperation with the N. E. A., has been held and it was one of the most successful that we have had both from the point of view of the fine programs and also the large attendance. The Hotel Vendome served as headquarters for the meeting. The total registration was 324. They came from 33 states, Washington, D. C., and Hawaii.

Various phases of the general theme, "What Lies Ahead for Mathematics" were discussed in seven sections. Discussion topics and the number attending each section follows:

Monday, June 30

1:45 P.M.	Joint Session with Department of Secondary Teachers	800 (Estimated)
3:15 P.M.	General Meeting.	160

Tuesday, July 1, 1941

12:00	Discussion Luncheon.	164
2:00 P.M.	Section I. What Lies Ahead for Arithmetic	90
2:00 P.M.	Section II. What Lies Ahead for Junior High School Mathematics?	120
2:00 P.M.	Section III. What Lies Ahead for Senior High School Mathematics?	160
4:00 P.M.	Demonstration of Multisensory Aids.	160

(See detailed program in the *Mathematics Teacher*, May, 1941, pp. 226–7.)

The Discussion Luncheon at which hosts and guests at nineteen tables talked over mathematical topics from Radio to Euclid, was held at the Hotel Vendome on Tuesday noon with Miss Dorothy Wheeler graciously presiding.

The entire program was prepared under the efficient direction of First Vice-President, Dr. F. Lynwood Wren, with the able assistance of the following program committees:

Arithmetic

Ben A. Sueltz, State Normal and Training School, Cortland, New York, Chairman

Foster E. Grossnickle, New Jersey State Teachers College, Jersey City, New Jersey

Guy M. Wilson, School of Education, Boston University, Boston

Junior High School

Edith Woolsey, Sanford Junior High School, Minneapolis, Minnesota, Chairman

Abner Bailey, Weeks Junior High School, Newton, Massachusetts

Senior High School

Martha Hildebrandt, Proviso Township High School, Maywood, Illinois, Chairman

Ralph Beatley, Graduate School of Education, Harvard University, Cambridge, Massachusetts

Multisensory Aids

E. H. C. Hildebrandt, Upper Montclair, New Jersey

Much credit for the success of the convention should be given to the Committee on Local Arrangements headed by Mr. Harold B. Garland of the High School of Commerce of Boston. Mr. Charles H. Mergendahl of Newtonville High School handled the publicity; Miss Lena G. Perrigo of Roxbury Memorial High School for Girls of Boston had charge of hospitality; other members of the local committee were: Professor Ralph Beatley of Harvard, Miss Margaret Cochran of the Senior

High School at Somerville, Professor Elmer B. Mode of Boston University, and Dr. Rolland R. Smith of Springfield.

Meeting of the Board of Directors

All officers and members of the Board of Directors and also all past presidents who were attending the convention were invited by the president, Miss Potter, to have dinner together on Monday, June 30 at the Hotel Vendome. After the dinner a business meeting was held.

Those present were Mary Potter, F. L. Wren, R. L. Morton, V. S. Mallory, A. Brown Miller, Dorothy Wheeler, William Betz, H. C. Christofferson, Martha Hildebrandt, Kenneth Brown, and Edith Woolsey.

Miss Potter asked Mr. Betz to introduce the topic for discussion: What can we do to keep mathematics alive in the schools? His first suggestion was that the Council publish some carefully constructed bulletins, to sell at cost, dealing with vital mathematical subjects, which could be used by both teachers and pupils.

His second suggestion was that the Council get some competent people to write articles to be published in the *Saturday Evening Post* or other popular magazines to arouse an interest in the subject of mathematics and to increase the appreciation of the value of the subject in the minds of the general public.

Mr. Wren moved that we endorse the suggestion as to the preparation of suitable publicity material on the importance of mathematics in modern life.

Mr. Miller seconded the motion. Following a discussion, Mr. Wren's motion was carried by a unanimous vote.

Miss Potter raised the question of how this publicity material should be handled.

Mr. Betz suggested that the president appoint a committee to study this thing and report at a later date.

Miss Potter announced that a friend of the Council, who wishes to be anonymous, has offered a prize of $75 for the best essay on logical thinking if the Council will put $75 with it.

After some discussion, Mr. Mallory moved that the president be empowered to appoint a committee to consider the awarding of a prize for a paper on reasoning in mathematics, and to report to the Council at the annual meeting.

Mr. Wren seconded the motion. The motion was carried by a unanimous vote.

The next topic for discussion was the time and place of the annual meeting.

It was moved by Mr. Morton and seconded by Mr. Wren that the president be empowered to make the decision as to the time and place of the annual meeting. The motion was passed by a unanimous vote.

It was moved, seconded, and carried that the meeting be adjourned.

Attendance at Boston Meeting

Alabama	1	Maine	15	Oregon	1
California	6	Massachusetts	162	Pennsylvania	4
Colorado	3	Michigan	4	Rhode Island	11
Connecticut	20	Minnesota	1	South Carolina	4
District of Columbia	4	Missouri	3	Tennessee	6
Florida	2	Nebraska	2	Texas	4
Georgia	1	New Hampshire	3	Utah	2
Illinois	5	New Jersey	17	Vermont	3
Indiana	3	New York	13	Washington	2
Iowa	3	North Carolina	1	Wisconsin	2
Kansas	1	Ohio	10	Honolulu	1
Kentucky	1	Oklahoma	3		324

IN OTHER PERIODICALS

By Nathan Lazar
The Bronx High School of Science, New York City

The American Mathematical Monthly

March 1941, Vol. 48, No. 3

1. Beiler, A. H. "A Peculiar Property of the Primitive Roots of 13," pp. 185–187.
2. Burington, R. S., "The Mil as an Angular Unit and Its Importance to the Army," pp. 188–189.
3. Georges, J. S. and Hedrick, E. R., "Mathematics Instruction for Purposes of General Education," pp. 189–197.
   A preliminary report of the representatives of mathematics on the special committee of the American Association for the Advancement of Science on the improvement of science teaching in colleges and universities.
4. Read, C. B., "Is a Mantissa Necessarily Positive?" pp. 203–204.

April 1941, Vol. 48, No. 4

1. Curtiss, D. R., "The Professional Interests of Mathematical Instructors in Junior Colleges," pp. 224–228.
2. Stewart, B. M., "Solitaire on a Checkerboard," pp. 228–233.
3. Gehman, H. M., "Complex Roots of a Polynomial Equation," pp. 237–239.
4. Zant, J. H., "A Course in Freshman Mathematics," pp. 246–250.
5. Bailey, R. P., "On the Treatment of Certain Problems of Elementary Probability," pp. 254–256.

National Mathematics Magazine

March 1941, Vol. 15, No. 6

1. Yates, Robert C., "The Trisection Problems," pp. 278–293. The third in a series of five chapters. It deals with mechanical trisectors of which the author describes fifteen. Diagrams and proofs are included.
2. Finkel, Benjamin F., "A History of American Mathematical Journals," (continued), pp. 294–302.
2. Capecelatro, Achille, "The Conic Functions," pp. 303–314.
4. Rickey, F. A., "Mathematics for Defense," p. 270.

April 1941, Vol. 15, No. 7

1. Saunders, S. T., "On The Applicative Phase of a Mathematical Principle," p. 330.
2. Finkel, Benjamin F., "A History of American Mathematical Journals," (continued), pp. 357–368.

School Science and Mathematics

April 1941, Vol. 41, No. 4

1. Downer, A. E., "Technical Mathematics in the Educational Program," pp. 316–321.
2. Clark, W. H., "Mathematics Offered to Commerce and Administration Students in Junior Colleges," pp. 340–345.
3. Harper, J. P. "Short Cuts and Approximations in Calculations," pp. 351–358.
4. Mansfield, Ralph, "A Note on the Rotation of Axes," pp. 378–379.
5. Hoyt, John P., "A Short Test for Geometry Teachers," p. 384.
6. "Recent Progress in Solid Geometry," p. 406. Résumé of an address delivered by Dr. Leo Zippin, of Queens (N. Y.) College, at the annual meeting of the American Mathematical Society.

May 1941, Vol. 41, No. 5

1. Thurber, Walter A., "Should We Invert the Fractional Divisor?" pp. 412–414.
2. Nyberg, Joseph A., "A List of Fundamental Theorems in Geometry." pp. 432–441.
3. Lange, Luise, "The Law $\sqrt{a}\cdot\sqrt{b} = \sqrt{ab}$ Holds for All Numbers," pp. 457–461.
4. Ballard, Ruth Mason, "The Teaching of Mathematics at the Junior College Level," pp. 482–486.

Scripta Mathematica

1940, Vol. 7, Nos. 1–4

1. Rees, Mina, "Lao Genevra Simons," pp. 7–8.
2. Fraenkel, A. A., "Natural Numbers as Ordinals," pp. 9–20.
3. Barbour, J. Murray, "Musical Logarithms," 21–31.
4. Lewis, May, "Formula for Calm" (a poem), p. 32.
5. Archibald, Ralph E., "Waring's Problem: Squares," pp. 33–48.
6. Bradley, A. Day, "The Mathematical Manuscripts in the Schwenkfelder Historical Library," pp. 49–58.
7. Higgins, Thomas James, "A Note on the

History of Mixed Partial Derivatives," pp. 59–62.

8. Karapetoff, Vladimir, "The Mathematical Thread in My Life," pp. 63–67.
9. Larguier, Everett H., "Brouwerian Philosophy of Mathematics," pp. 69–78.
10. Infeld, Leopold, "The Fourth Dimension and Relativity," pp. 79–85.
11. Weismiller, Edward, "Mathematician" (a poem), p. 86.
12. Polacheck, Harry, "The Structure of the Honeycomb," pp. 87–98.
13. Hill, Lester S., "Notes on the Regular Icosahedron and the Regular Dodecahedron," pp. 99–109.
14. Fréchet, Maurice, "On Some Contributions to the Foundations of the Calculus of Probability," pp. 110–112.
15. "Mathematics in Nature," facing p. 112. (photographs of snow crystals).
16. Boyd, Rutherford, "Mathematical Themes in Ornament," facing p. 112 and p. 143.
17. "Algebraic Ornaments," facing p. 112.
18. Feld, J. M., "Counterpoints and Associated Cubic Curves," pp. 113–118.
19. McCoy, John Calvin, "The Anatomy of Magic Squares," (continued), pp. 143–153.
20. Moritz, Robert E., "Memorabilia Mathematica," pp. 153–155.
21. Meyer, Jerrome S., "A Two Inch Line with a Six Inch Perimeter," pp. 156–157.
22. Curiosa, p. 68, pp. 157–159.

Miscellaneous

1. "Arithmetic Seatwork for Review of Testing; for Intermediate Grades," *Grade Teacher*, 58: 40+, March, 1941.
2. Aten, H. D., "Functional Geometry," *Curriculum Journal*, 12: 23–25, January, 1941.
3. Braverman, Benjamin, "Handling the Repeater in First Term Geometry," *High Points*, 23: 43–47, February, 1941.
4. Breslich, E. R., "Place of Mathematics in Education for Social Change," *School Review*, 49: 104–113, February, 1941.
5. Deans, E., "Contribution of Grouping to Number Development," *Childhood Education*, 17: 307–310. March, 1941.
6. Dimmitt, R., "Modern Objective Tests: Addition and Subtraction for Primary Grades," *Grade Teacher*, 58: 62, March, 1941.
7. Dwyer, G. W., "March Arithmetic Test," *Instructor*, 50: 19, March, 1941.
8. Eastman, M. C., "Arithmetic Review for Upper Intermediate and Grammar Grades," *Grade Teacher*, 58: 63–64, March, 1941.
9. Epstein, P. S., "Secondary School Mathematics in Relation to College Physics," *American Journal of Physics*, 9: 34–37, February, 1941.
10. Gibbons, C. C., "Predictive Value of the Most Valid Items of an Examination," *Journal of Educational Psychology*, 31: 616–621, November, 1940.
11. Hart, R. W., "Terminal Courses in Mathematics," *Junior College Journal*, 11: 253–256, January, 1941.
12. Hartung, M. L., "Concerning Improvement in Mathematics Instruction," *Chicago Schools Journal*, 22: 125–127, January, 1941.
13. Hawkins, G. E., "Selected References on Secondary-School Instruction Mathematics," *School Review*, 49: 143–147, February, 1941.
14. Morse, M., "Training in Scientific Theory," *Science* (new series) 93: 6 (supplement), February 28, 1941.
15. Rosenberg, R. R., "Capable Business Arithmetic Teachers Make Bookkeeping Instruction Easy," *National Business Education Quarterly*, 9: 19–22+, December, 1940.
16. Rourke, E. K., "Enrichment of High School Mathematics," *School* (Secondary Edition). 29: 552–556, February, 1941.
17. Stadtlander, E. J., "Arithmetic Theories, a State Course of Study, and Text Books," *Elementary School Journal*, 41: 438–453, February, 1941.
18. "Two Examples of Study Groups," *Elementary School Journal*, 41: 406–408, February, 1941.
19. Waynen, M. Van, and Reasoner, W. H., "General Mathematics for the Ninth Grade," *California Journal of Secondary Education*, 16: 116–119, February, 1941.
20. Wegner, F. R., "Arithmetic Skills: How Our Work is Assigned and Evaluated," *Grade Teacher*, 58: 60–61+, April, 1941.
21. Winhold, E. P., "Securing and Maintaining Interest in Teaching Mathematics," *School*, (Secondary Edition), 29: 557–559, February, 1941.

New Books Received

November 1940 to March 1941

GRAPHS: HOW TO MAKE AND USE THEM, by Herbert Arkin and Raymond P. Colton. Pp. xvii+236. 1940. Harper and Brothers. Price $3.00.

BASIC GEOMETRY, by George David Birkhoff and Ralph Beatley. Pp. 294. 1941. Scott, Foresman and Company. Price $1.32.

PLANE TRIGONOMETRY, Revised Edition, by Raymond W. Brink. Pp. xii+110. 1940. D. Appleton-Century Company, New York. Price $2.00.

INTRODUCTION TO THE THEORY OF EQUATIONS, by Nelson Bush Conkwright. Pp. viii+214. 1941. Ginn and Company. Price $2.00.

GEOMETRY FOR TODAY, by Alexander J. Cook, based on "A Junior Geometry" by A. W. Siddons and R. T. Hughes. Pp. vii+260. 1940. Macmillan Company, Toronto, Price $1.00.

COLLEGE ALGEBRA, by H. T. Davis. Pp. xiii+423. 1940. Prentice-Hall, Inc. Price $2.50.

NEW VOCATIONAL MATHEMATICS FOR BOYS, by William H. Dooley and David Kriegel. Pp. xi+349. 1941. D. C. Heath and Company. Price $1.64.

AN INTRODUCTION TO DIFFERENTIAL GEOMETRY, by Luther Pfahler Eisenhart. Pp. x+304. 1940. Princeton University Press. Price $3.50.

A GUIDE TO A FUNCTIONAL PROGRAM IN THE SECONDARY SCHOOL. State Department of Education, Florida Program for Improvement of Schools, Bulletin No. 10. October 1940.

FUNDAMENTAL MATHEMATICS, by Duncan Harkin. Pp. xv+434. 1941. Prentice-Hall, Inc. Price $3.00.

ESSENTIALS OF ALGEBRA, FIRST COURSE, by W. W. Hart. Pp. vii+439. 1941. D. C. Heath and Company. Price $1.28.

ESSENTIALS OF HIGH SCHOOL ALGEBRA, by W. W. Hart. Pp. ix+582. 1941. Price $1.60.

PLANE GEOMETRY, by Howard B. Kingsbury and R. R. Wallace. Pp. xi+484. 1941. Bruce Publishing Company. Price $1.68.

MATHEMATICS FOR EVERYDAY AFFAIRS, by Virgil S. Mallory. Pp. vii+471. 1940. Benjamin H. Sanborn and Company, Chicago and Boston. Price $1.28.

MATHEMATICS INSTRUCTION IN THE UNIVERSITY HIGH SCHOOL, by members of the Department of Mathematics of the University High School of the University of Chicago. Publications of the Laboratory Schools of the University of Chicago. No. 8, November 1940. Price $1.00.

INTRODUCTORY COLLEGE MATHEMATICS, Revised Edition, by W. E. Milne and David R. Davis. Pp. xvi+438+ii+82. Ginn and Company. 1941. Price $3.00.

PRACTICAL MATHEMATICS, Part III, GEOMETRY WITH APPLICATIONS, by Claude Irwin Palmer and Samuel F. Bibb. Pp. xii+206. 1941. McGraw-Hill Book Company. Price $1.25.

TREATISE ON ALGEBRA, by George Peacock. Vol. I, pp. xiv+309; Vol. II, pp. x+455. 1940. (Reprinted from the 1842 edition) Scripta Mathematica, Yeshiva College, New York. Price $6.50.

PLANE GEOMETRY, by F. Eugene Seymour and Paul James Smith. Pp. xi+467. 1941. The Macmillan Company, New York. Price $1.60.

ARITHMETIC IN GRADES I AND II: A Critical Summary of New and Previously Reported Research, by Wm. A. Brownell, with the assistance of Roy A. Doty and Wm. C. Rein. Pp. xi+175. 1941. Duke University Research Studies in Education No. 6. Price $1.50.

BRIEF TRIGONOMETRY, by Arthur E. Crathorne and Gerald E. Moore. Pp. v+121. 1941. Henry Holt and Company. Price $1.20.

CREDIT PROBLEMS OF FAMILIES. Pp. vii+99. 1941. Vocational Division, Bulletin #206, Home Economics Series #23, Office of Education, United States Dept. of Interior. Price 20¢.

ESSENTIALS OF ALGEBRA, second course, by Walter W. Hart. Pp. vii+344. 1941. D. C. Heath and Company. Price $1.32.

SOLID GEOMETRY, by Henry L. C. Leighton. Pp. iii+108. 1941. Edwards Brothers, Ann Arbor, Michigan.

PRACTICAL MATHEMATICS, PART II, ALGEBRA WITH APPLICATIONS, 4th edition, by Claude Irwin Palmer and Samuel Fletcher Bibb. Pp. xiii+256. 1941. McGraw-Hill Book Company. Price $1.25.

BOOKKEEPING AND ACCOUNTING, INTRODUCTORY COURSE, fourth edition, by Arthur Henry Rosenkampf and William Carroll Wallace.

Pp. xix+349. 1941. Prentice-Hall, Inc., Price $1.62.

MEASUREMENT IN TODAY'S SCHOOLS, by C. C. Ross. Pp. xviii+595. 1941. Prentice-Hall, Inc.

THE TEACHING OF ARITHMETIC TO LOW-ABILITY STUDENTS IN THE ELEMENTARY SCHOOLS, by Henry L. Smith and Merrill T. Eaton. Pp. 127. November, 1940. Bulletin of the School of Education, Indiana University, Bloomington, Indiana. Price 50¢.

MODERN TREND GEOMETRY, by William W. Strader and Lawrence D. Rhoads. Pp. xi+444. 1941. The John C. Winston Company.

American Education Week 1941

"EDUCATION FOR A STRONG AMERICA" is the highly appropriate theme of the twenty-first annual observance of American Education Week, November 9–15, 1941. The daily topics are:

Sunday, November 9	—Seeking World Order
Monday, November 10	—Building Physical Fitness
Tuesday, November 11	—Strengthening National Morale
Wednesday, November 12	—Improving Economic Wellbeing
Thursday, November 13	—Safeguarding School Support
Friday, November 14	—Learning the Ways of Democracy
Saturday, November 15	—Enriching Family Life

If the schools are to be safeguarded in a period when taxes and the cost of living are skyrocketing, it is vitally important that every opportunity to interpret the work of the schools be utilized. This hour in our national life is critical. Effective democratic education is imperative if we are to meet the issues of our time as a free nation. This message must be taken to all the people for consideration and action.

One of our best opportunities to seek public understanding and appreciation of the schools and the place of education in our nation is during American Education Week. The National Education Association has prepared materials to assist you in planning to make the observance successful in your school system, your individual school, and your own classroom.

Special packets are available for the following school levels: kindergarten-primary grades, elementary (grades 4, 5, and 6) junior high school, and high school. Each packet contains a classroom supply of posters, leaflets, and stickers, a special 32-page manual for the proper school level, a folder for the Sunday observance, and other materials.

New features also available this year include (1) a two-color button to be worn home by pupils carrying the slogan "Visit Your Schools American Education Week," (2) two musical plays—one for high school use and one for elementary school use—both written especially for the occasion by Jean Byers, author of the noted production "On Our Way" prepared for the Educational Policies Commission, (3) a $1\frac{1}{2}$ minute 35 mm. sound movie trailer for use in commercial theaters just before and during American Education Week featuring Lowell Thomas and entitled "Education for a Strong America."

Address the National Education Association, 1201 Sixteenth Street, N.W., Washington, D.C., for complete information and prices.

Eighth December Meeting of the National Council of Teachers of Mathematics

December 31, 1941-January 1, 1942, Lehigh University Bethlehem, Pennsylvania

PROGRAM

General Program Chairman, R. L. Morton, Ohio University, Athens, Ohio

WEDNESDAY, DECEMBER 31, 1941

10:00 a.m. General Session
Christmas Saucon Hall, Room 205
Presiding: F. Lynwood Wren, Peabody College, Nashville Tennessee
General Mathematics Curriculum Coöperatives
J. I. Baugher, Superintendent of Schools, Hershey, Pennsylvania
Mathematics in the General Curriculum
Stevenson W. Fletcher, Jr., George School, George School, Pennsylvania
Practical Air Navigation
Hale Pickett, State Teachers College, West Chester, Pennsylvania
High School Mathematics in the Defense Program
Carl N. Shuster, State Teachers College, Trenton, New Jersey

1:00 p.m. Multi-Sensory Aids, Demonstrations
Chemistry Building, Room 244
Presiding: E. H. C. Hildebrandt, State Teachers College, Montclair, New Jersey
1:00 Stereoscopic Mapping from the Air
1:10 A Film on Higher Mathematics
1:20 A Triple Integral
Produced by E. A. Whitman, Carnegie Institute of Technology, Pittsburgh, Pennsylvania
1:35 Trivision and Mathematics, A demonstration by the inventor of trivision film, Douglas F. Winnek, Photographic Engineer, Mount Vernon, New York
1:55 Recent Film Strips

2:30 p.m. Junior High School Section
Christmas Saucon Hall, Room 109
Presiding: R. Reed Edwards, Smedley Junior High School, Chester, Pennsylvania
New Objectives for Ninth Grade Mathematics
Wm. Willits, Southern Junior High School, Reading, Pennsylvania
The In-Service Training of Mathematics Teachers
Nanette L. Roche, Supervisor of Mathematics, Baltimore, Maryland
Approximate Computation
Carl N. Shuster, State Teachers College, Trenton, New Jersey

2:30 p.m. Senior High School Section
Christmas Saucon Hall, Room 205
Presiding: Virgil Mallory, State Teachers College, Montclair, New Jersey
The Foundation Stones of the Number System
H. W. Brinkmann, Swarthmore College, Swarthmore, Pennsylvania
The Coming Revolution in Mathematics
C. O. Oakley, Haverford College, Haverford, Pennsylvania

2:30 p.m. Teacher Training Section
Christmas Saucon Hall, Room 103
Presiding: L. H. Whitcraft, Ball State Teachers College, Muncie, Indiana
The Extra-Technical Training of Mathematics Teachers
Arnold Dresden, Swarthmore College, Swarthmore, Pennsylvania
In-Service Training of Teachers
Elon G. Salisbury, State Teachers College, California, Pennsylvania
The Training Mathematics Teachers Should Have to Meet the Social and Business Problems of today.
W. S. Schlauch, New York University, New York City
The Responsibility of the Mathematics Teacher in Curriculum Building
J. W. Faust, State Teachers College, Mount Pleasant, Michigan
Discussion

6:30 p.m. Joint Dinner with the Association and the Society at the Hotel Bethlehem

THURSDAY, JANUARY 1, 1942

10:00 a.m. General Session
Christmas Saucon Hall, Room 205
Presiding: R. L. Morton, Ohio University, Athens, Ohio
An Interesting Application of Continued Fractions
C. R. Atherton, Hershey Junior College, Hershey, Pennsylvania
What the Engineering School Wants in High School Mathematics
Tomlinson Fort, Lehigh University, Bethlehem, Pennsylvania

1:00 p.m. Multi-Sensory Aids, Demonstrations
Chemistry Building, Room 244
Presiding: E. H. C. Hildebrandt, States Teachers College, Montclair, New Jersey
1:00 Reproduction of Radio Broadcast
1:15 A Problem in Modern Mathematics
Produced by Sister M. Leonida, Decatur, Indiana
1:35 East High School Goes to the Mathematics Exhibit
Produced under the direction of H. W. Charlesworth, East High School, Denver, Colorado
1:45 Demonstration of Three-Dimensional Projection
Slides by the inventor, Pompey Mainardi, Newark College of Engineering, Newark, New Jersey

2:30 p.m. Senior High School Section
Christmas Saucon Hall, Room 205
Presiding: Mary Potter, Supervisor of Mathematics, Racine, Wisconsin
Broadening Horizons in the Teaching of Secondary Mathematics
Margaret Rae Davis, Ashley High School, Ashley, Pennsylvania
Some Misconceptions as to the Nature of Indirect Proof in Geometry
Nathan Lazar, High School of Science, New York, New York
Mathematics as an Interpretation of Life and the World
W. S. Schlauch, New York University, New York City

2:30 p.m. Teacher Training Section
Christmas Saucon Hall, Room 103
Presiding: C. R. Atherton, Hershey Junior College, Hershey, Pennsylvania
Current Requirements and Standards for the Preparation of Mathematics Teachers
Panel Discussion

2:30 p.m. Multi-Sensory Aids Section
Christmas Saucon Hall, Room 109
Presiding: E. H. C. Hildebrandt, State Teachers College, Montclair, New Jersey
Home-Made Mathematical Instruments Constructed at the University of Vermont
G. H. Nicholson, University of Vermont, Burlington, Vermont
Graphical Representation of Complex Roots
Howard F. Fehr, State Teachers College, Montclair, New Jersey
Demonstrations of Mathematical Relationships and Applications With Moving Apparatus
Phillip S. Jones, The Edison Institute, Dearborn, Michigan

Program Committees

Junior High School

Edith Woolsey, Minneapolis, Minn, Chairman

Senior High School

Martha Hildebrandt, Maywood, Ill., Chairman

Teacher Training

L. H. Whitcraft, Muncie, Ind., Chairman

Multi-Sensory Aids

E. H. C. Hildebrandt, Montclair, N.J., Chairman

Exhibits

Luther J. Deck, Muhlenberg College, Allentown, Pennsylvania, Chairman
Earle H. Weinsheimer, Allentown, Pennsylvania
Paul K. Whytock, Bethlehem, Pennsylvania
Ethel H. Hicks, Reading, Pennsylvania
Louise E. Uhler, Easton, Pennsylvania

Housing

Mrs. Laura McD. Ashbaugh, Bethlehem, Pennsylvania, Chairman
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Truman L. Koehler, Muhlenberg College, Allentown, Pennsylvania, Chairman
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Wm. S Sterner, Springfield, New Jersey
Harold P. Thomas, Bethlehem, Pennsylvania

Mathematics Exhibit: Christmas Saucon Hall, Room 209